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EEN320 - Power Systems I (Συστήματα Ισχύος I)

Part 3: The power transformer

https://sps.cut.ac.cy/courses/een320/

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Last updated: January 14, 2025

Learning objectives

After this part of the lecture and additional reading, you should be able to . . .

1. . . . explain the role of power transformers in power systems;
2. . . . explain the fundamental principles of single-phase and three-phase power transformers;
3. . . . derive idealised and practical models for power transformers;
4. . . . determine stationary operating conditions of power transformers via circuit calculations.

Outline

1 Outline

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1. **Why do we need power transformers?**
2. Single-phase transformer
3. Three-phase transformer

1 Why do we need power transformers?

- Power transformers are essential pieces of equipment in power systems
- They are used to
  - Step-up voltage, e.g. at terminals of generators (step-up transformer)
  - Step-down voltage, e.g. to distribute power at low voltage to end-user (step-down transformer)
  - Control voltages at some busbars (in sub-transmission and distribution networks)
  - Control power flows in some parts of meshed networks

1 Why do we need high voltage levels? (1)

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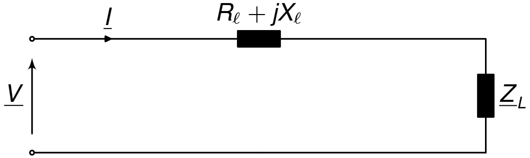

1) Total transmitted apparent power: $\underline{S} = \underline{V}\,\underline{I}^*$

Active power losses due to line resistance: $P_{\text{loss}} = I^2 R_\ell$

- $\Rightarrow$ Lower current to transmit same power at higher voltage
  - Less transmission losses
  - Fewer conductors and paralleled systems needed
  - But: higher costs for insulators and power poles
- $\Rightarrow$ Economic optimum $V_{\text{economic}} \sim \sqrt{|\underline{S}|}$ ($\underline{S}$ is apparent power)

  Voltage magnitude (in kV) should increase approximately with square root of magnitude of transmitted apparent power S (in MVA)

1 Why do we need high voltage levels? (2)

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2) Surge impedance loading (SIL) or natural loading (*details in the last chapter of this course!*)

- SIL: loading of a line at which reactive power is neither produced nor absorbed

$$\text{SIL} = \frac{V^2}{Z_W}$$

- Characteristic impedance of transmission line $Z_W \approx \sqrt{\frac{L'}{C'}}$
- L' is line inductance per unit length, C' is line capacitance per unit length
- Loaded below SIL: line ”supplies” reactive power to system; consequence: voltage raises (Ferranti effect)
- Loaded above SIL: line ”consumes” reactive power; consequence: voltage reduces
- For typical transmission line $Z_W \approx 250 \ldots 380\ \Omega$

$\Rightarrow$ $V = \sqrt{SIL \cdot Z_W}$

$$V \approx 16 \ldots 19\sqrt{SIL}$$

- Note: this is a similar relation to 1)

1 Why do we need high voltage levels? (3)

3) Provision of short circuit current (*details in EEN442!*)

- Usual overcurrent protection devices require certain min. short circuit current to trigger
- Short circuit current

  $$I_{SC} \approx \frac{V}{\sqrt{3}X} \qquad (1/\sqrt{3} \text{ since we need to express } V \text{ in line-ground}),$$

  where X is short circuit reactance of considered circuit
- Seems to indicate $I_{SC} \uparrow$ if $V \uparrow$
- But: for synchronous machines and transformers, usually *relative* reactance x instead of absolute reactance X given
- Relation between x and X defined via nominal impedance

  $$Z_N = \frac{V_N}{\sqrt{3}I_N}$$

  of machine/transformer, i.e.,

  $$X = xZ_N = x\frac{V_N}{\sqrt{3}I_N} = x\frac{V_N^2}{S_N}$$

1 Why do we need high voltage levels? (4)

3) Provision of short circuit current (ctd.)

- Hence, short circuit current (with $V = V_N$)

$$I_{SC} = \frac{V_N}{\sqrt{3}X} = \frac{V_N}{\sqrt{3}} \frac{S_N}{xV_N^2} = \frac{S_N}{x\sqrt{3}V_N} \approx \frac{1}{V_N}$$

$\Rightarrow$ Actually, for given nominal (rated) power S_N, $I_{SC} \downarrow$ if $V_N \uparrow$

- Likewise, to keep I_{SC} fixed, one needs that $V_N \uparrow$ if $S_N \uparrow$
- Note: these considerations are only valid for one single machine/transformer
- Transmission line impedances are not severely modified by voltage level
- But: higher voltage level $\Rightarrow$ longer lines $\Rightarrow$ higher absolute lumped line impedance $\Rightarrow$ lower I_{SC} for fixed V

4) Interconnection of several grids

2 Outline

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- From electromagnetism[1], we know that Maxwell's equations connect the magnetic field quantities with the electric field and the electric current intensity.
- More specifically, Ampère's law states that:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J} + \frac{\partial(\epsilon \boldsymbol{E})}{\partial t}$$

where:

- ***H*** is the magnetic field intensity vector (ένταση μαγνητικού πεδίου) and is measured in Ampere per meter (A/m)
- ***J*** is the electric current density (πυκνότητα ηλεκτρικού ρεύματος) and is measured in ampere per square meter (A/m^2)
- ***E*** is the electric field density (ένταση ηλεκτρικού πεδίου)
- ϵ is the dielectric constant of the material (διηλεκτρική σταθερά υλικού)

[1] Check Physics III and Electromagnetism course

- Under the assumption that our system size is much smaller than the electromagnetic wavelength $\frac{c}{f} = \frac{2\pi c}{\omega}$, or equivalently, when:

$$\frac{\omega \ell}{c} << 1$$

where c is the speed of light, f the frequency, $\omega = 2\pi f$ is the angular frequency, and ℓ is the largest distance in our system.

- Then, the electromagnetic waves propagate instantaneously and the electric and magnetic fields are decoupled. Thus, we can analyse the permanent magnetic field (μόνιμο μαγνητικό πεδίο) which simplifies to:

$$\nabla \times \boldsymbol{H} = \boldsymbol{J}$$

- Note that the magnetic flux density (μαγνητική επαγωγή) is given by:

$$\boldsymbol{B} = \mu \boldsymbol{H}$$

  - μ is the magnetic permeability of the material (μαγνητική διαπερατότητα υλικού), which can be defined also as $\mu = \mu_r \mu_0$ with $\mu_0 = 4\pi \cdot 10^{-7}$ the vacuum permeability and μ_r the relative permeability. The SI unit is Henry per meter (H/m).
  - $\boldsymbol{B}$ is measured in Tesla (T) which is equal to Weber per meter squared (Wb/m^2).

- Using Stokes theorem, we can get the integral form of Ampère's law:

$$\oint_C \boldsymbol{H}_{tan} dl = I_{enclosed}$$

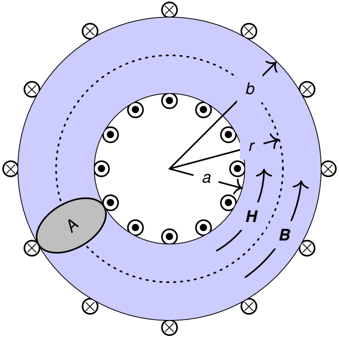
showing that the tangential component of the magnetic field intensity vector ($\boldsymbol{H}_{tan}$) integrated along a closed path (C) equals the net current enclosed by that path ($I_{enclosed}$).

2.1 Electromagnetic Concepts - (Very) Brief review of Faraday's law

- Faraday's law: A changing magnetic field (example: magnet moving through a conducting coil) generates an electric field and vice-versa
- Fundamental principle for many electrical appliances, such as motors, generators or *transformers*
- Mathematically:

$$\mathcal{E}(t) = \frac{d\Phi}{dt}$$

  - $\mathcal{E}(t)$ is electromotive (ηλεκτρεγερτική) force (EMF) = voltage induced
  - $\Phi = \boldsymbol{A}\boldsymbol{B}_{tan}$ is the magnetic flux (πεπλεγμένη ροή) = cross-sectional area exposed to magnetic field $\times$ component of magnetic field vector normal to that area. Φ is measured in Webers (Wb).



Using Ampère's law, we get:

- For $r > b \rightarrow \oint_C \boldsymbol{H}_{tan} dl = NI - NI = 0$
- For $r < a \rightarrow \oint_C \boldsymbol{H}_{tan} dl = 0$
- For $a < r < b \rightarrow \oint_C \boldsymbol{H}_{tan} dl = 2\pi r H = NI = \mathcal{F}$
  where $\mathcal{F}$ is called magnetomotive force (mmf) (μαγνητεγερτική δύναμη) and the SI units are *ampere-turn* (αμπερελίγματα).

2.1 Magnetic circuits

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- Due to symmetry, we get:

$$H = \frac{\mathcal{F}}{2\pi r}$$

- Which leads to:

$$B = \frac{\mu \mathcal{F}}{2\pi r}$$

- If the cross-section is small, we can assume uniform magnetic field equal to:

$$B_\mu = \frac{\mu \mathcal{F}}{2\pi r_\mu}$$

where $r_\mu = (a+b)/2$

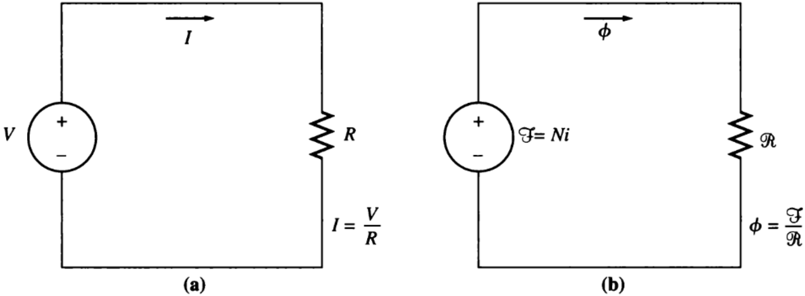
- Leading to magnetic flux of:

$$\Phi = B_\mu A = \mathcal{F}\frac{\mu A}{\ell_\mu} = \frac{\mathcal{F}}{\mathcal{R}_m} \qquad \Rightarrow \qquad \mathcal{F} = \Phi \mathcal{R}_m$$

where $\ell_\mu = 2\pi r_\mu$ is the mean length of the magnetic circuit and $\mathcal{R}_m = \frac{\ell_\mu}{\mu A}$ the reluctance of the magnetic circuit.

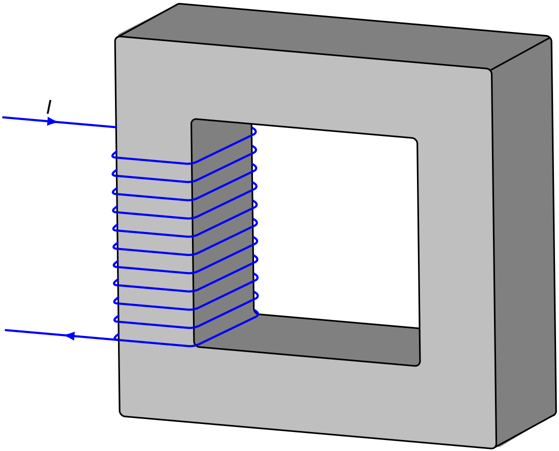
2.1 Magnetic circuits

This way, we can transform the problem into an equivalent electric circuit:



Chapman, S.J. (2012). Electric machinery fundamentals (5e). McGraw-Hill.

2.1 Magnetic circuits - Example



Task. In the above magnetic circuit, compute the current I at the coil with $N = 500$ so that the magnetic field is $B = 1$ T. It is given that the outer dimensions of the magnetic core are 20cmx20cm and the cross-section area is 4cmx4cm. The relative magnetic permeability is $\mu_r = 3980$ and can be considered constant.

Solution.

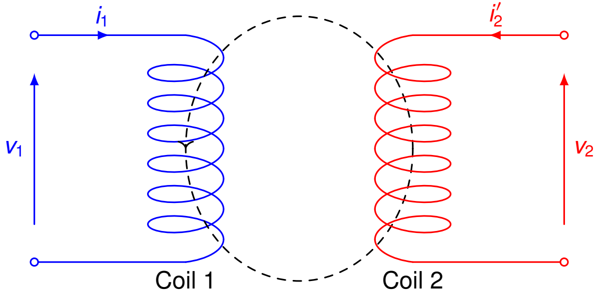
$$\ell_\mu = 4 \cdot 16 = 64cm = 0.64m$$

$$A = 4 \cdot 4 = 16cm^2 = 16 \cdot 10^{-4} m^2$$

$$\mathcal{F} = \Phi \frac{\ell_\mu}{\mu_0 \mu_r A} = BA \frac{\ell_\mu}{\mu_0 \mu_r A} = B \frac{\ell_\mu}{\mu_0 \mu_r} = 127.95 \,\text{ampere-turns}$$

$$I = \frac{\mathcal{F}}{N} = 0.256\,A$$

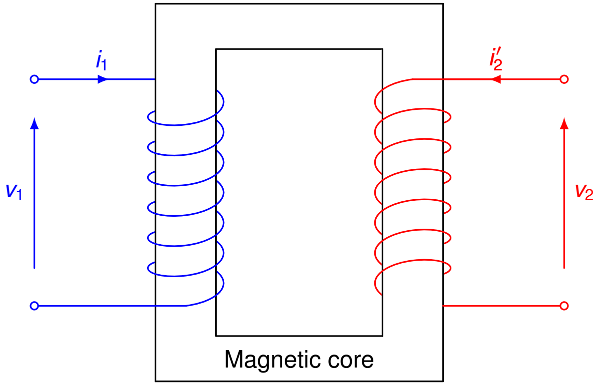
2.1 Principle of a transformer - Coupled coils (1)



- Alternating current in coil produces alternating magnetic field (Faraday's law)
- Two coils located in a common magnetic field influence each other

→ Magnetic coupling: any variation of current in one coil causes variation of the flux linked with adjacent coil

2.1 Principle of a transformer - Coupled coils (2)



- Two coils mounted on common magnetic core (usually iron) → magnetic flux in both coils almost identical (strong magnetic coupling)

2.1 Principle of a transformer - Coupled coils (3)

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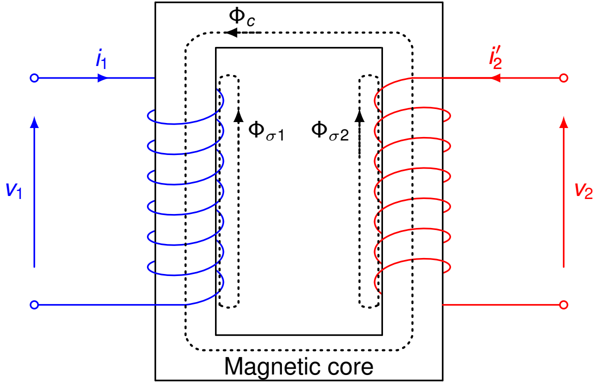


- AC voltage v_1 at terminals of coil 1 $\rightarrow$ AC current i_1 in coil 1 $\rightarrow$ alternating magnetic field $\rightarrow$ voltage induced in coil 2 $\rightarrow$ current i_2' in coil 2 $\rightarrow$ magnetic field superposed to field caused by i_1
- Φ_c mutual flux in magnetic core flowing through both coils; $\Phi_{\sigma 1}$ and $\Phi_{\sigma 2}$ are leakage fluxes; note: above drawing assumes $|i_1| > |i_2'|$

- Mutual flux (ροή μαγνητίσεως) Φ_c: flux crossing both coils and causes magnetic coupling
- In each coil a *leakage flux* (ροή σκέδασης) $\Phi_{\sigma 1}$ and $\Phi_{\sigma 2}$ is developed in addition to mutual flux Φ_c.
- Leakage flux: components of magnetic field crossing coil 1 or 2 but not passing through magnetic core.
- Usually: $\Phi_{\sigma 1} \ll \Phi_c$, $\Phi_{\sigma 2} \ll \Phi_c$
- Flux linkages (συνολική ροή) ψ_1 and ψ_2 in coils given by

$$\psi_1 = N_1 \Phi_c + \psi_{\sigma 1}$$
$$\psi_2 = N_2 \Phi_c + \psi_{\sigma 2}$$

- If we apply Ampère's law on the figure of slide 22, we get:

$$N_1 i_1 - N_2 i_2' = H_c \ell_c = \frac{\ell_c B_c}{\mu_c} = \left(\frac{\ell_c}{\mu_c A_c} \right) \Phi_c = \mathcal{R}_m \Phi_c$$

- ℓ_c: the length of the path in the core
- A_c: the cross-section area of the magnetic core
- μ_c: the magnetic permeability of the magnetic core
- $\mathcal{R}_m$: core reluctance (μαγνητική αντίσταση) of magnetic circuit
- Direction of magnetic fluxes dependent on sense of winding of coils (here is a minus)

2.2 Ideal transformer model - Assumptions

Ideal transformer model is based on following assumptions

- A1: There are no leakage fluxes $\rightarrow$ ideal coupling between coils
- A2: There are no losses in the transformer (neither in core nor in windings)
- A3: The permeability of the core material is infinite (= core has ideal magnetic conductivity)

2.2 Ideal transformer model - Relation between primary and secondary voltages

- A1: No leakage fluxes, i.e. $\psi_{\sigma 1} = \psi_{\sigma 2} = 0$

→ Faraday's law of induction (Slide 14) implies that

$$v_1 = \frac{d\psi_1}{dt} = N_1 \frac{d\Phi_c}{dt}$$
$$v_2 = \frac{d\psi_2}{dt} = N_2 \frac{d\Phi_c}{dt}$$

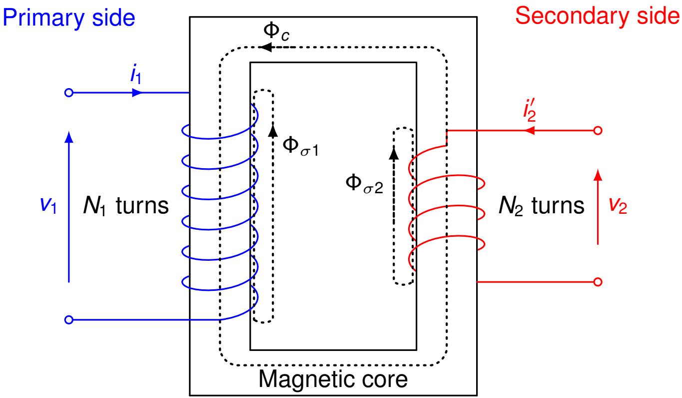
- Rearranging terms

$$\frac{v_1}{N_1} = \frac{d\Phi_c}{dt} = \frac{v_2}{N_2}$$

→ Turns ratio between primary and secondary voltage

$$\boxed{\frac{v_1}{v_2} = \frac{N_1}{N_2} = c \in \mathbb{R}}$$

(Note: in three-phase transformers c can also be a complex number)



- Number of turns can be different on primary and secondary windings
- Step-up transformer: secondary voltage $v_2 > $ primary voltage v_1
- Step-down transformer: secondary voltage $v_2 < $ primary voltage v_1

2.2 Ideal transformer model - Relation between primary and secondary currents

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- A2: No losses (the coils have no resistance)
- A3: ideally conducting magnetic core (the permeability of the core material is infinite) $\mu_c \rightarrow \infty \rightarrow \mathcal{R}_m = 0$

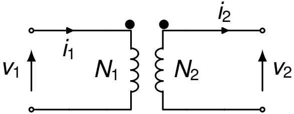
→ We obtain from Slide 24

$$N_1 i_1 - N_2 i_2' = 0$$

- Define $i_2 = -i_2'$ to obtain

$$\boxed{\frac{i_2}{i_1} = \frac{N_1}{N_2} = c \in \mathbb{R}}$$

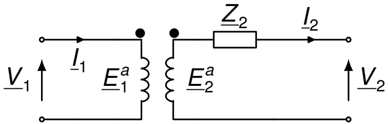
2.2 Ideal transformer model - Schematic representation

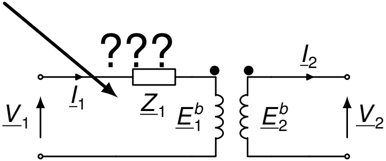


$$\frac{v_1}{v_2} = \frac{N_1}{N_2} = c, \qquad \frac{i_2}{i_1} = \frac{N_1}{N_2} = c$$

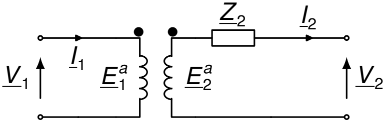
- Single-phase two-winding transformer
- c is the turns ratio
- "•" indicate sense of winding of coils:
  "•" opposite of each other $\rightarrow$ identical direction

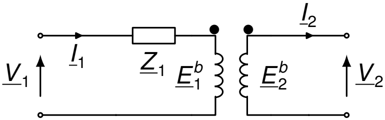
2.2 Ideal transformer model - Impedance transformation





2.2 Ideal transformer model - Impedance transformation





- Apply KVL

$$\underline{V}_1 = \underline{E}_1^a = c\underline{E}_2^a = c(\underline{V}_2 + \underline{I}_2\underline{Z}_2)$$

$$\underline{V}_2 = \underline{E}_2^b = \frac{1}{c}\underline{E}_1^b = \frac{1}{c}(\underline{V}_1 - \underline{I}_1\underline{Z}_1)$$

- Solve equations for $\underline{V}_2$

$$c\underline{V}_2 = \underline{V}_1 - c\underline{I}_2\underline{Z}_2$$
$$c\underline{V}_2 = \underline{V}_1 - \underline{I}_1\underline{Z}_1$$

- Subtract equations from each other

$$0 = -c\underline{I}_2\underline{Z}_2 + \underline{I}_1\underline{Z}_1$$

$$\boxed{\frac{\underline{Z}_1}{\underline{Z}_2} = c\frac{\underline{I}_2}{\underline{I}_1} = c^2}$$

2.2 Ideal transformer model - Summary

- Relation of primary and secondary voltages $\underline{E}_1$ and $\underline{E}_2$

$$\boxed{\frac{\underline{E}_1}{\underline{E}_2} = c}$$

- Relation of primary and secondary currents $\underline{I}_1$ and $\underline{I}_2$

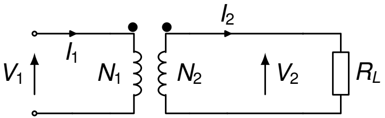
$$\boxed{\frac{\underline{I}_1}{\underline{I}_2} = \frac{1}{c}}$$

- Relation of primary and secondary impedances $\underline{Z}_1$ and $\underline{Z}_2$

$$\boxed{\frac{\underline{Z}_1}{\underline{Z}_2} = c^2}$$

- There are no losses in an idealised transformer!

2.2 Ideal transformer model - Example



Task. A single-phase transformer has 2500 turns on the primary winding and 500 turns on the secondary winding. The resistive load on the secondary side is $\underline{Z}_L = R_L = 12\ \Omega$. The voltage magnitude (RMS) at the terminals of the primary winding is $V_1 = 6$ kV. Determine the voltage magnitude (RMS) at the secondary side and the load current.

Solution.

$$c = \frac{N_1}{N_2} = \frac{2500}{500} = 5$$

$$V_2 = \frac{V_1}{c} = 1.2\ \text{kV}$$

$$I_2 = \frac{V_2}{R_L} = 0.1\ \text{kA}$$

- Thus far: Discussion and derivation of idealised transformer model under *idealised* conditions and assumptions
- In the following, we will drop some of these assumptions to obtain a more realistic transformer model with
  - Non-zero leakage fluxes
  - Winding losses
  - Finite core permeability μ_c

2.3 Real transformer model - Leakage fluxes

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- Each linkage flux $\psi_{\sigma 1}$ and $\psi_{\sigma 2}$ only interacts with one coil

→ Can consider them independently of each other and as being caused by additional separate *leakage inductors* with inductances $L_{\sigma 1}$ and $L_{\sigma 2}$ in primary, respectively secondary, circuit

- Flux linkage between leakage inductors and leakage fields happens over air

→ Linear relationship

$$\psi_{\sigma 1} = L_{\sigma 1} i_1$$
$$\psi_{\sigma 2} = L_{\sigma 2} i_2'$$

- Leakage fluxes diminish coupling between coils 1 and 2

→ They represent a *non-ideal* property of a transformer

2.3 Real transformer model - Winding losses

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- Real windings possess ohmic (resistive) component
- R_1 resistance of coil 1; R_2 resistance of coil 2
- KVL for primary circuit

$$v_1 = R_1 i_1 + \frac{d\psi_1}{dt} = R_1 i_1 + \frac{d}{dt}\left(\psi_{\sigma 1} + N_1 \Phi_c\right)$$
$$= R_1 i_1 + L_{\sigma 1}\frac{di_1}{dt} + N_1 \frac{d\Phi_c}{dt}$$

- KVL for secondary circuit

$$v_2 = R_2 i_2' + \frac{d\psi_2}{dt} = R_2 i_2' + \frac{d}{dt}\left(\psi_{\sigma 2} + N_2 \Phi_c\right)$$
$$= R_2 i_2' + L_{\sigma 2}\frac{di_2'}{dt} + N_2 \frac{d\Phi_c}{dt}$$
$$= -R_2 i_2 - L_{\sigma 2}\frac{di_2}{dt} + N_2 \frac{d\Phi_c}{dt}$$

2.3 Real transformer model - Core losses

- Ohmic law of magnetic circuit from Slide 24 with $\mathcal{R}_m \neq 0$

$$\mathcal{R}_m \Phi_c = N_1 i_1 - N_2 i_2'$$

- Calculation of no load losses $\rightarrow$ open-circuit the secondary $\rightarrow$ set $i_2' = 0$

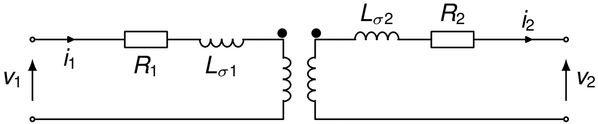
$\rightarrow$ Magnetising current

$$i_1\Big|_{i_2'=0} = i_m = \frac{\mathcal{R}_m \Phi_c}{N_1}$$

- Magnetising current i_m can be considered in transformer model via magnetising inductance L_h
- Value of L_h can be derived from equation for induced voltage

$$N_1 \frac{d\Phi_c}{dt} = L_h \frac{di_m}{dt}$$

2.3 Real transformer model with leakage inductors and winding resistors



- Real windings possess ohmic (resistive) component
- R_1 resistance of coil 1; R_2 resistance of coil 2
- Real windings possess a component of leakage flux that only interacts with one coil, but does not link to the other coil

→ Can consider this by additional separate *leakage inductors* with inductances $L_{\sigma 1}$ and $L_{\sigma 2}$ in primary, respectively secondary, circuit

- $L_{\sigma 1}$ leakage inductance coil 1; $L_{\sigma 2}$ leakage inductance coil 2

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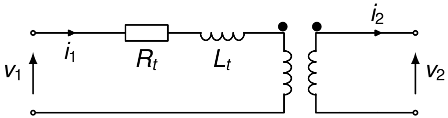


- Transform resistances and inductances to primary side

$$R_t = R_1 + c^2 R_2$$
$$L_t = L_{\sigma 1} + c^2 L_{\sigma 2}$$

→ Practical transformer model with flux leakages and ohmic losses in windings

2.3 Real transformer model - Core losses

- In reality, magnetic core has voltage- and frequency-dependent losses

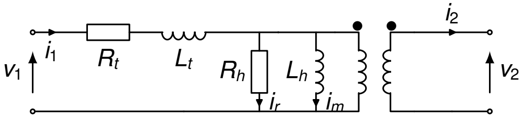
→ Magnetic conductivity of core also not ideal

- Power delivered on secondary side is lower than power injected on primary side!

→ Transformer also consumes power if there is no current flowing on the secondary side ($i_2 = 0$) to compensate for losses in magnetic core

- These losses are called *no load losses*
- No load losses = losses on primary coil + losses in magnetic core
- Reactive component of losses can be modelled by *magnetising inductance*
- Magnetising inductance then absorbs *magnetising current*

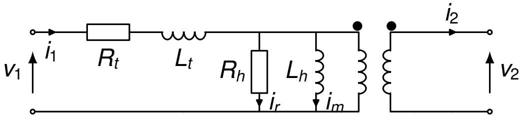
2.3 Real transformer model - Core losses



- In addition to reactive power losses there are also active power losses in core
- Active power losses represented by resistance R_h in parallel to L_h
- Primary current under load (i.e. $i_2 \neq 0$)

$$i_1 = i_m + i_r + \frac{i_2}{c}$$

- In practice, $i_m + i_r < 0.01 i_{2,\text{rated}}$



- Complete transformer model with
  - Leakage losses (represented by L_t)
  - Coil losses (represented by R_t)
  - Core losses (represented by L_h (inductive) and R_h (ohmic))
- Further phenomena to improve model
  - Core saturation $\rightarrow$ non-linear behaviour
  - Inrush current (energising current: non-sinusoidal and large DC component)
  - Non-sinusoidal exciting current

- Usual relation between leakage inductance L_t and magnetising inductance L_h

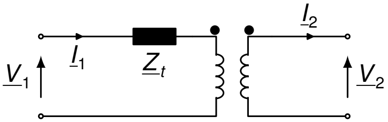
$$L_h \gg L_t$$

- Usual relation between coil resistance R_t and core resistance R_h

$$R_h \gg R_t$$

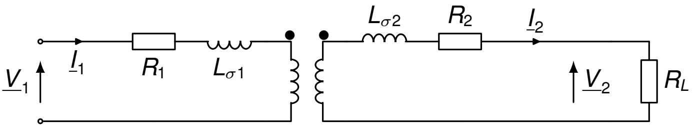
→ Neglect R_h and L_h in model

- Be careful: this simplification is only valid under load!



$$\underline{Z}_t = R_t + j\omega L_t$$

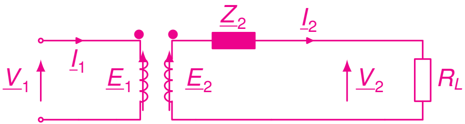
2.3 Real transformer model - Example (Task)



Task. A single-phase transformer has 2000 turns on the primary winding and 500 turns on the secondary winding. The winding resistances are $R_1 = 2.0\ \Omega$ and $R_2 = 0.125\ \Omega$. The leakage reactances are $X_1 = \omega L_{\sigma_1} = 8.0\ \Omega$ and $X_2 = \omega L_{\sigma_2} = 0.5\ \Omega$. The resistive load on the secondary side is $\underline{Z}_L = R_L = 12\ \Omega$. The voltage magnitude (RMS) at the terminals of the primary winding is $V_1 = 6$ kV. Determine the voltage at the load and the load current.

Solution.

Equivalent circuit with all quantities referred to secondary side



Express all quantities with respect to secondary side

$$c = \frac{N_1}{N_2} = \frac{2000}{500} = 4 \qquad \underline{Z}_2 = R_2 + jX_2 + \frac{R_1 + jX_1}{c^2} = 0.25 + j1.0\ \Omega$$

$$\underline{E}_1 = \underline{V}_1 = 6\underline{/0^\circ}\ \text{kV} \qquad \underline{E}_2 = \frac{\underline{E}_1}{c} = \frac{\underline{V}_1}{c} = 1.25\ \text{kV}$$

$$\underline{I}_2 = \frac{\underline{E}_2}{\underline{Z}_2 + R_L} = 0.1022 - j0.0084 = 0.1\underline{/-4.7^\circ}\ \text{kA}$$

$$\underline{V}_2 = R_L\underline{I}_2 = 1.2\underline{/-4.7^\circ}\ \text{kV}$$

A single-phase two-winding transformer is rated 20 kVA, 480/120 V, 60 Hz.

- During a short-circuit test, where rated current at rated frequency is applied to the 480-volt winding (denoted winding 1), with the 120-volt winding (winding 2) shorted, the following readings are obtained: $V_1 = 35$ V, $P_1 = 300$ W.
- During an open-circuit test, where rated voltage is applied to winding 2, with winding 1 open, the following readings are obtained: $I_2 = 12$ A, $P_2 = 200$ W.

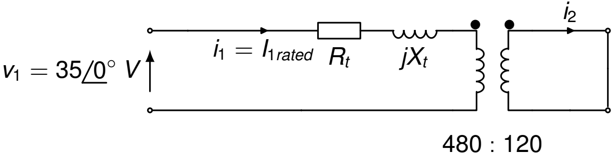
Task.

1. From the short-circuit test, determine the equivalent series impedance $Z_t = R_t + jX_t$ referred to winding 1. Neglect the shunt admittance.
2. From the open-circuit test, determine the shunt admittance $Y_m = G_c + jB_m = \frac{1}{Z_h} = \frac{1}{R_h||jX_h}$ referred to winding 1. Neglect the series impedance.

Source: J. D. Glover, M. S. Sarma and T. Overbye, ”Power System Analysis & Design”, 6th edition, Cengage Learning, 2017

2.3 Transformer short-circuit and open-circuit tests - Example

Solution. 1) Short-circuit neglecting admittance



$$I_{1\,rated} = \frac{S_{rated}}{V_{1\,rated}} = \frac{20 \cdot 10^3}{480} = 41.667\ A$$

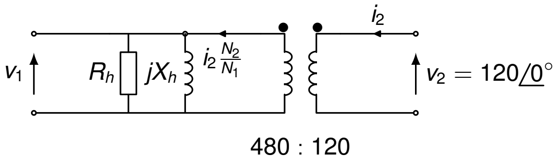
$$R_t = \frac{P_1}{I^2_{1\,rated}} = \frac{300}{41.667^2} = 0.1728\,\Omega$$

$$|Z_t| = \frac{v_1}{I_{1\,rated}} = \frac{35}{41.667} = 0.84\,\Omega$$

$$X_t = \sqrt{Z_t^2 - R_t^2} = 0.822\,\Omega$$

$$Z_t = R_t + jX_t = 0.1728 + j0.822 = 0.84\angle 78.13^\circ\ \Omega$$

Solution. 2) Open-circuit neglecting impedance



$$v_1 = \frac{N1}{N2} v_2 = \frac{480}{120} 120 = 480\ V$$

$$G_c = \frac{P_2}{v_1^2} = \frac{200}{480^2} = 0.000868\ S$$

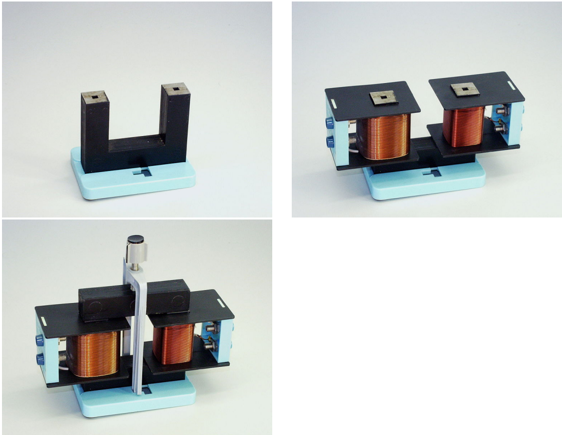
$$|Y_m| = \frac{i_1}{v_1} = \frac{i_2 \frac{N_2}{N_1}}{v_1} = \frac{12 \frac{120}{480}}{480} = 0.00625\ S$$

$$B_m = \sqrt{Y_m^2 - G_c^2} = 0.00619\ S$$

$$Y_m = 0.000868 - j0.00619 = 0.00625\angle -82.02^\circ\ S$$

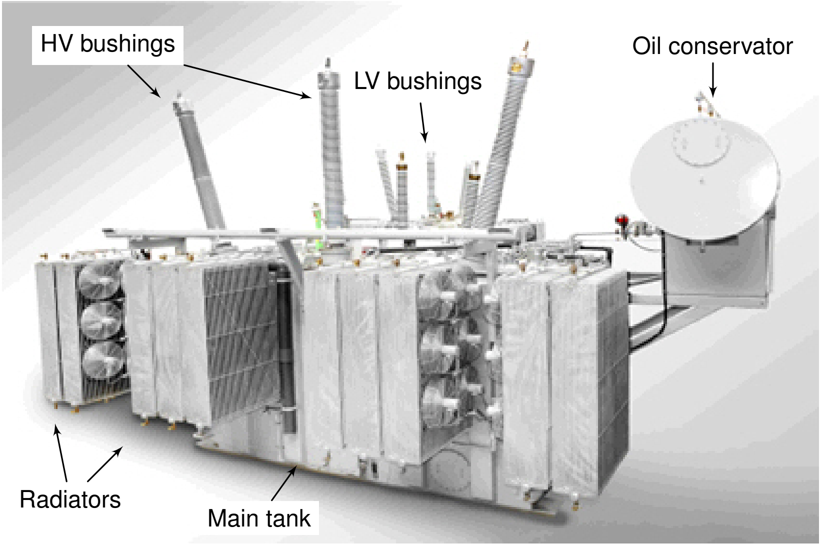
Why is B_m negative?

3 Basic single-phase transformer



Single-phase transformer

3.1 Schematic representation of a three-phase transformer



3.1 Main components of a three-phase transformer

- HV and LV bushings make electrical connections from external transmission or distribution circuits into transformer tank
- Primary and secondary coils are normally made of copper, with paper insulation and are mounted on the outside of magnetic steel core
- Magnetic core is formed from laminated sheets of steel to minimise electrical losses, which would occur if a solid steel core was used
- Core and windings are mounted within a steel tank, which is sealed against atmospheric pollution, particularly moisture
- Tank is oil-filled, with oil acting as both high voltage insulation material and cooling medium
- Conservator tank provides a ”head” of oil to ensure that tank is full, even under expansion and contraction of oil when thermally cycled during service
- External radiators are connected to tank to cool oil

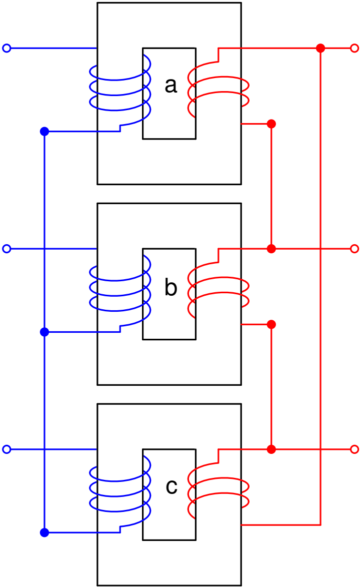
3.1 Example of a three-phase transformer (1)

Large generator or transmission transformer can have mass of 300 – 500 tonnes $\rightarrow$ can represent significant logistical challenge in transporting unit from factory, via road, rail or ship, to its final destination at (potentially remote) substation

3.1 Example of a three-phase transformer (2)

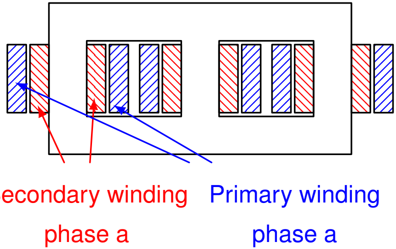
©Chris Lyles

3.2 Three-phase transformer - First type

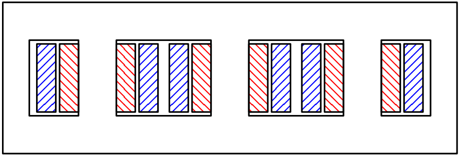


- Use three single-phase transformers
- If one transformer fails, only that one needs to be replaced
- Easier to carry
- Usually used for transformers of very large nominal power
- Phases are not coupled magnetically

Core-type construction (3 legs/limbs)



Shell-type construction (5 legs/limbs)

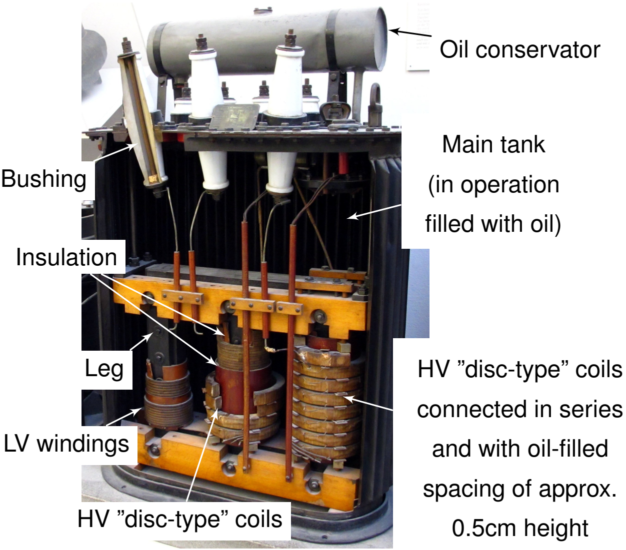


- Three phases mounted on single magnetic (iron) core
- Lighther and more compact than first type; less material and space needed
- Phases are magnetically coupled
- In balanced operation: fluxes in legs sum up to zero

$$\Phi_a + \Phi_b + \Phi_c = 0$$

- Core configuration or shell configuration

3.2 Cross-sectional scheme of three-phase transformer



©Dmm2va7

3.2 Example of a core-type three-phase transformer

Cyprus University of Technology

3.2 Example of a three-phase transformer

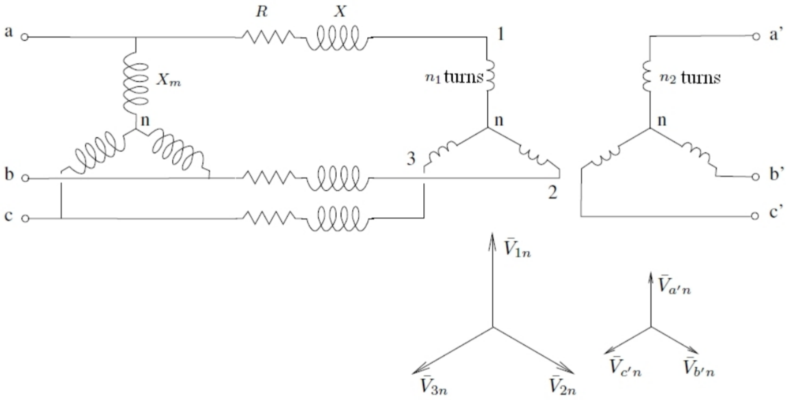
©Nikitas Lamprou

3.2 Example of a three-phase transformer

3.2 Example of a three-phase transformer

Four main different configuration possibilities:Y-Y, Y-Delta, Delta-Y, Delta-Delta

1. Y-Y-configuration



- Preferred at very high voltage level since voltage across each coil is $\sqrt{3}$ lower
- Possibility to connect neutral to ground (safety protection)

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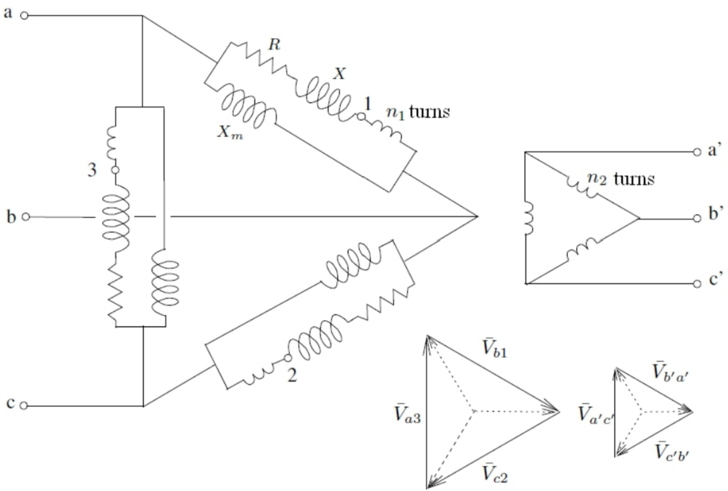
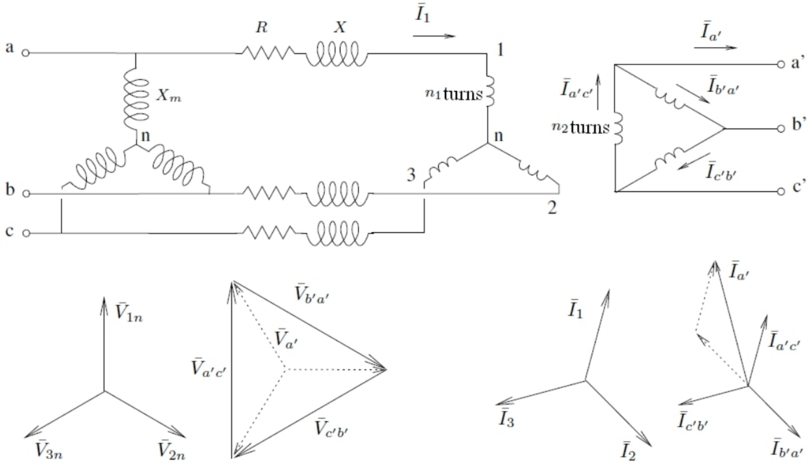
2. Delta-Delta-configuration



- Preferred for high currents, as currents in phases $\sqrt{3}$ lower
- Also used to eliminate harmonics

3.3 Configuration of three-phase transformers (3)

3. Y-Delta-configuration



- Frequent transformer configuration connecting generator to grid
- On high-voltage side, neutral point is grounded (for protection)

3.3 Three-phase transformer - Classification

- Standardized abbreviation of I.E.C. (International Electrotechnical Commission)
- Classification of transformer configuration consists of 2 letters and 1 integer
- First letter: Configuration of high-voltage side; upper-case letter used (**Y** or **D**)
- Second letter: Configuration of low-voltage side; lower-case letter used (**y** or **d**)
- Integer: Phase shift between voltages at primary and secondary winding of the same phase as multiple of 30° ($\pi/6$) assuming the transformer is ideal
- Additionally in Y-connection: **n** after **Y** or **y** to indicate that neutral is grounded

- Example 1: Yd5
  - High-voltage side in Y-connection; neutral not grounded
  - Low-voltage side in Delta-connection
  - Phase-shift between high- and low-voltage: $5 \cdot 30^\circ = 150^\circ$
- Example 2: Yny0
  - High-voltage side in Y-connection; neutral grounded
  - Low-voltage side in y-connection
  - Phase-shift between high- and low-voltage: 0°

  $\rightarrow$ If same configuration on high- and low-voltage side, then no phase displacement between voltages or displacement of 180° (depending on sense of winding coils)

3.4 Three-phase transformer - Transformation ratio (1)

- In balanced operation, in principle can use single-phase equivalent circuit for analysis
- But if analysing Yd or Dy connections, need to consider
  1. Not same voltages on primary and secondary side: one has phase voltages the other line voltages

  → Transformation ratio also affected!

  2. Phase and line voltages also differ in phase

  → Additional phase displacement Phase displacement is integer multiple of $\pi/6 = 30^\circ$

3.4 Three-phase transformer - Transformation ratio (2)

- Impact on amplitude considered by introducing additional scalar k
  - Yy or Dd: $k = 1$
  - Yd-configuration: $k = \sqrt{3}$
  - Dy-configuration: $k = \frac{1}{\sqrt{3}}$
- Impact on phase displacement considered by introducing additional phase shift element

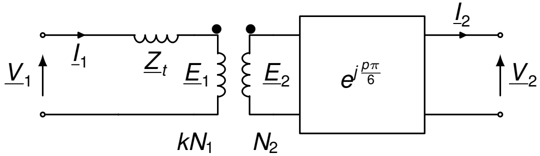
$$e^{j\frac{p\pi}{6}}$$

where $p = \{0, 1 \ldots, 11\}$ is an integer

→ Complex transformation ratio

$$\boxed{\underline{c} = k\frac{N_1}{N_2}e^{j\frac{p\pi}{6}}}$$

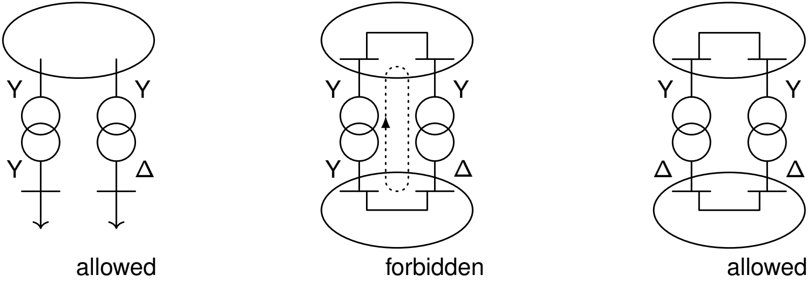
- Note: $|e^{j\frac{p\pi}{6}}| \rightarrow$ amplitude of transformation not influenced by p



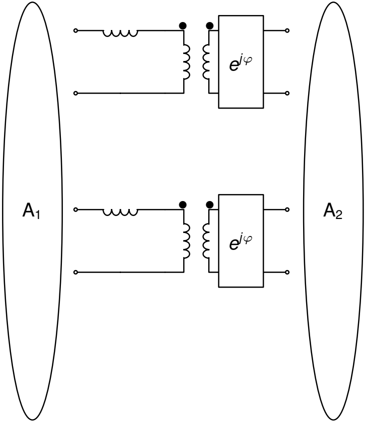
- Combine single-phase transformer model with complex transformation ratio
- Then

$$\underline{E}_1 = \underline{c}\,\underline{V}_2$$
$$\underline{I}_2 = \underline{c}^*\,\underline{I}_1$$

3.4 Three-phase transformer - Caution with phase displacements



- Many transformers present in electrical network
- If they work in parallel (i.e. in one same loop), they have to have the same phase displacement p
- If this is not taken into account, there might be very large power flows between transformers



- Two transformers in parallel with same phase displacement
  $\varphi = \varphi_A = \varphi_B$
- Then, phase displacement elements can be removed without changing
  - Magnitude of branch currents and bus voltages
  - Complex power flows in branches (as $\underline{V}\,\underline{I}^* = \underline{V}e^{j\varphi}\underline{I}^* e^{-j\varphi}$)

→ Usually, phase displacement elements ignored in analysis and computations under balanced steady-state conditions

3.4 Summary

- Power systems work at different voltage levels
- Transformers needed to connect different voltage levels
- Models for single- and three-phase transformer
- Idealised and practical (more complicated) model
- In practice, core losses (shunt impedances) can often be neglected (under load!)
- Three-phase transformers have in general a complex transformation ratio
- Voltage ratio of transformer can be modified whilst it is in service by adding device known as an On-line Tap-changer (OLTC); this allows different numbers of turns to be connected on one of the coils (on each phase) of the transformer; typical adjustment in range $\pm 10\%$