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EEN320 - Power Systems I (Συστήματα Ισχύος I)

Part 5: Introduction to rotating machines

https://sps.cut.ac.cy/courses/een320/

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Today's learning objectives

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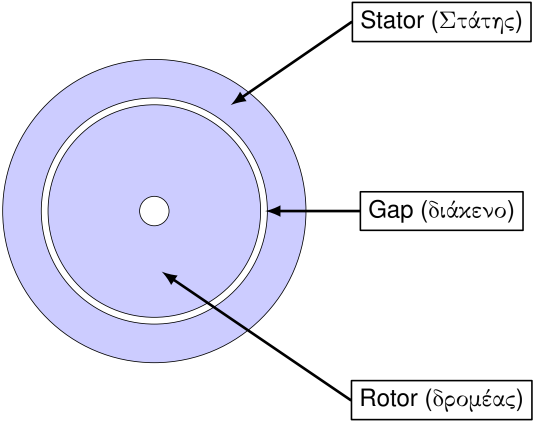
After this part of the lecture and additional reading, you should be able to . . .

1. . . . explain the basic principles of electromechanical energy conversion;
2. . . . explain the fundamental principles of rotating machines.

Outline

1 Outline

1 Basic rotating machine components



1 Lorentz Force Law review

From electromagnetics, we know that Lorentz Force Law:

$$\underline{F} = q(\underline{E} + \underline{v} \times \underline{B})$$

where:

- $\underline{F}$ is the force (newtons) on a particle of charge q (coulombs) in the presence of electric and magnetic fields
- $\underline{E}$ is the electric field in volts per meter
- $\underline{B}$ is the magnetic field in teslas
- $\underline{v}$ is the velocity of the particle q relative to the magnetic field, in meters per second.

1 Lorentz Force Law review

Ignoring the electric field:

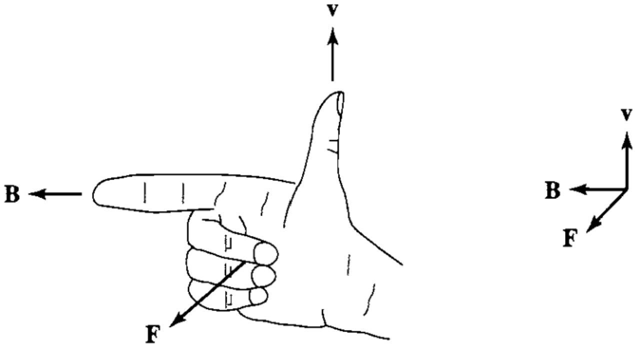


Figure 3.1 Right-hand rule for determining the direction magnetic-field component of the Lorentz force $\mathbf{F} = q(\mathbf{v} \times \mathbf{B})$.

Fitzgerald, A. E., Kingsley, C., & Umans, S. D. (2003). Electric machinery. McGraw-Hill.

1 Application of Lorentz Force Law on a rotor

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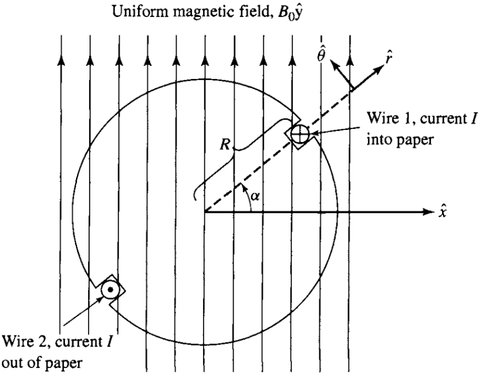
A non-magnetic rotor (δρομέας) containing a single-turn coil is placed in a uniform magnetic field of magnitude B_0 generated by the stator (see later), as shown below. The coil sides are at radius R and the wire carries current I. Find the θ-directed torque as a function of rotor position. Assume that the rotor is of length ℓ.



Adapted from "Fitzgerald, A. E., Kingsley, C., & Umans, S. D. (2003). Electric machinery. McGraw-Hill".

1 Application of Lorentz Force Law on a rotor

The force per unit length (in N) acting on the wire is given by

$$\underline{F} = \underline{I} \times \underline{B} \text{ N}$$

For wire of length ℓ and current I is given as:

$$F = -IB_0\ell \sin(\alpha)$$

For two wires:

$$F = -2IB_0\ell \sin(\alpha)$$

The total torque (in Nm) is then:

$$T = -2IRB_0\ell \sin(\alpha)$$

And, if we assume a rotation $\alpha = \omega t$:

$$T = -2IRB_0\ell \sin(\omega t)$$

1 Application of Lorentz Force Law on a rotor

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Since there is a current flowing, the rotor also produces a magnetic field. The magnetic flux density B_R generated by the rotor due to the current I is:

$$B_R = \mu H_R = \frac{\mu I}{G}$$

where G depends on the geometry of the rotor loop. For a circular one then $G = 2R$. For a rectangular one, G depends on the length-to-width ratio.
So, we get:

$$T = \frac{AG}{\mu} B_R B_0 \sin(\alpha)$$

where $A = 2R\ell$ is the area of the wire on the rotor if assumed rectangular.
We can rewrite as:

$$\boxed{T = k B_R B_0 \sin(\alpha) = k \underline{B}_R \times \underline{B}_0}$$

where $k = AG/\mu$ is a factor depending on the machine construction.

Conceptual explanation:

1. A magnetic north and south poles can be associated with the stator and rotor of a machine due to the current flows;
2. Similar to a compass needle trying to align with the earth's magnetic field, these two sets of fields attempt to align;
3. If one of the fields (stator or rotor) rotates, the other ones tries to "catch up". Torque is associated with their displacement from alignment:
   - In a motor, the stator magnetic field rotates ahead of that of the rotor, "pulling" on it and performing work
   - In a generator, the rotor magnetic field rotates ahead of that of the stator, "pulling" on it and performing work

1 Basic rotating machines principle

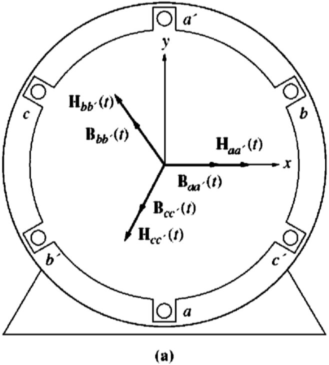
The torque in a real machine, depends on:

1. The strength of the rotor magnetic field;
2. The strength of the external (stator) magnetic field;
3. The $\sin$ of the angle between them; and,
4. A constant depending on the construction of the machine.

2 Outline

2 Three-phase machine stator

Assume now a three-phase stator (στάτης) with windings aa', bb', cc' as shown in the figure below:



Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

2 Three-phase machine stator: magnetic field

We feed the three coils with (in Ampere):

$$I_{aa'}(t) = I_M \sin(\omega t)$$
$$I_{bb'}(t) = I_M \sin(\omega t - 120^\circ)$$
$$I_{cc'}(t) = I_M \sin(\omega t - 240^\circ)$$

Which generates magnetic field intensity (in Ampere-turns/m):

$$\underline{H}_{aa'}(t) = H_M \sin(\omega t) \underline{/0^\circ}$$
$$\underline{H}_{bb'}(t) = H_M \sin(\omega t - 120^\circ) \underline{/120^\circ}$$
$$\underline{H}_{cc'}(t) = H_M \sin(\omega t - 240^\circ) \underline{/240^\circ}$$

- The direction of the field is shown on the figure and given by the ”right-hand rule”. The phase shown at the end is the spacial degrees.
- The magnitude changes sinusoidally but direction same.

2 Three-phase machine stator: magnetic field

The flux density is given by $\underline{B} = \mu \underline{H}$ (in Tesla):

$$\underline{B}_{aa'}(t) = B_M \sin(\omega t)\underline{\angle 0^\circ}$$
$$\underline{B}_{bb'}(t) = B_M \sin(\omega t - 120^\circ)\underline{\angle 120^\circ}$$
$$\underline{B}_{cc'}(t) = B_M \sin(\omega t - 240^\circ)\underline{\angle 240^\circ}$$

where $B_M = \mu H_M$.

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Examples:

$\omega t = 0^\circ$

$$\begin{aligned}\underline{B}_{net} &= \underline{B}_{aa'} + \underline{B}_{bb'} + \underline{B}_{cc'} \\ &= 0 + \left(-\frac{\sqrt{3}}{2}B_M\right)\underline{/120^\circ} + \left(\frac{\sqrt{3}}{2}B_M\right)\underline{/240^\circ} \\ &= 1.5B_M\underline{/-90^\circ}\end{aligned}$$

$\omega t = 90^\circ$

$$\begin{aligned}\underline{B}_{net} &= \underline{B}_{aa'} + \underline{B}_{bb'} + \underline{B}_{cc'} \\ &= B_M\underline{/0^\circ} + \left(-\frac{1}{2}B_M\right)\underline{/120^\circ} + \left(-\frac{1}{2}B_M\right)\underline{/240^\circ} \\ &= 1.5B_M\underline{/0^\circ}\end{aligned}$$

2 Three-phase machine stator: magnetic field

In the general case[1]:

$$\underline{B}_{net} = 1.5 B_M \left(\sin(\omega t)\underline{\hat{x}} - \cos(\omega t)\underline{\hat{y}} \right)$$

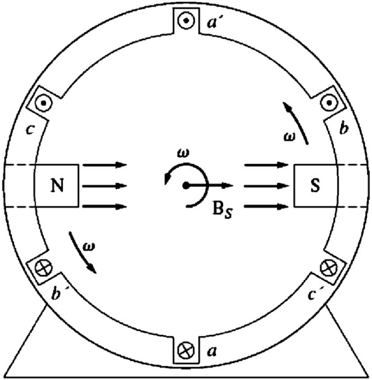
How about changing the rotation of the field?
$\rightarrow$ We swap the current in two of the phases:

$$\underline{B}_{net} = 1.5 B_M \left(\sin(\omega t)\underline{\hat{x}} + \cos(\omega t)\underline{\hat{y}} \right)$$

[1] Try to prove this using the trigonometric relations used in part 2.

2 Three-phase machine stator: two-pole

This is equivalent to a two-pole (north-south) field rotating:



The magnetic poles complete **one** full *mechanical* rotation for every **one** *electrical* cycle:

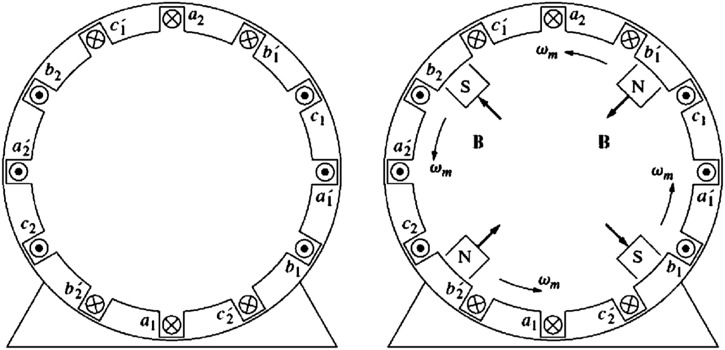
$$f_e = f_m$$

$$\omega_e = \omega_m$$

Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

2 Three-phase machine stator: four-pole

This is equivalent to **two** two-pole (north-south) field rotating:



The magnetic poles complete **one** full *mechanical* rotation for every **two** *electrical* cycle:

$$\theta_e = 2\theta_m, \quad f_e = 2f_m, \quad \omega_e = 2\omega_m$$

Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

2 Three-phase machine stator: *P*-pole

In general for a *P*-poles machine:

$$\boxed{\theta_e = \frac{P}{2}\theta_m} \text{ (rad)}$$

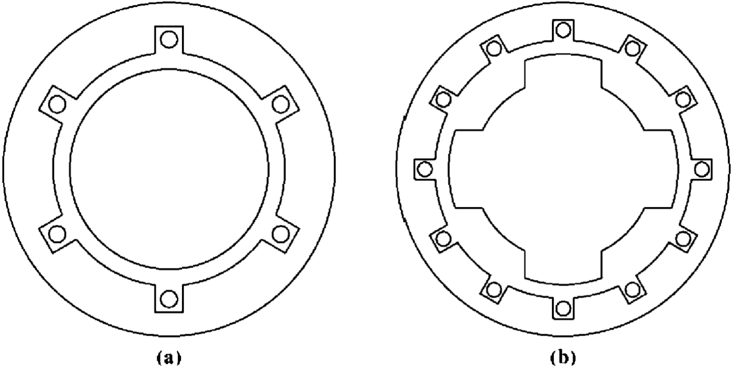
$$\boxed{f_e = \frac{P}{2}f_m} \text{ (Hz)}$$

$$\boxed{\omega_e = \frac{P}{2}\omega_m} \text{ (rad/s)}$$

$$\boxed{n = \frac{120 f_e}{P}} \text{ (rounds per minute)}$$

2 Three-phase machine rotor type

In general, there are two type of rotors: (a) cylindrical or nonsalient-pole (κυλινδρικός δρομέας) (b) salient-pole (έκτυπους πόλους).

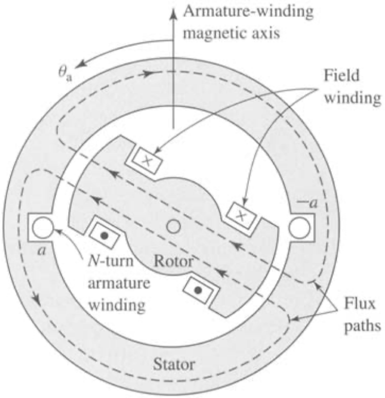


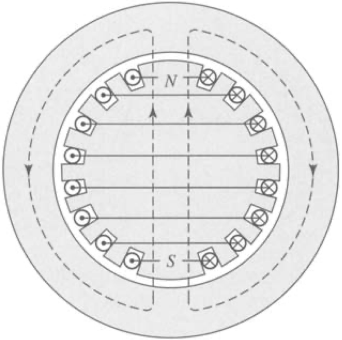
Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

2 Three-phase machine rotor type

- Salient-pole is characteristic of hydroelectric generators because hydraulic turbines operate at lower speeds
  $\rightarrow$ lower speeds requires higher number of poles
  $\rightarrow$ salient poles are better mechanically for large number of poles.
- Steam and gas turbines operate better at high speeds and are commonly two- or four-pole cylindrical-rotor.

2 Three-phase machine rotor type





Salient pole field windings create the magnetic field. The construction of the poles generates a sinusoidal field.

Cylindrical rotor needs an uneven distribution of the conductors to generate a sinusoidal magnetic field.

Fitzgerald, A. E., Kingsley, C., & Umans, S. D. (2003). Electric machinery. McGraw-Hill.

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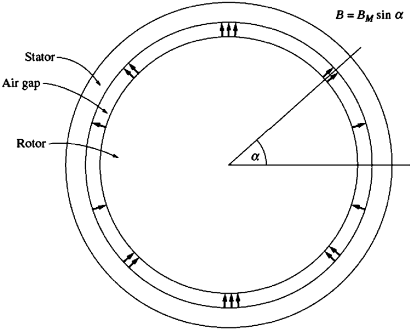
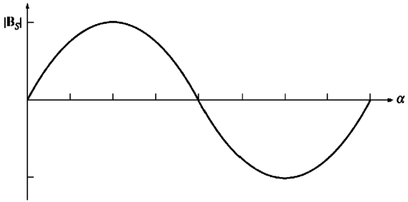


The conductors on the surface of the cylindrical rotor should be distributed as $n_C = N_C \cos(\alpha)$ with N_C the number of conductors at 0° (maximum).



If we freeze the rotor and we observe the magnetic field generated by it at an angle α, it will be sinusoidal:

$$B = B_M \sin(\alpha)$$

Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

2 Three-phase machine rotor supply

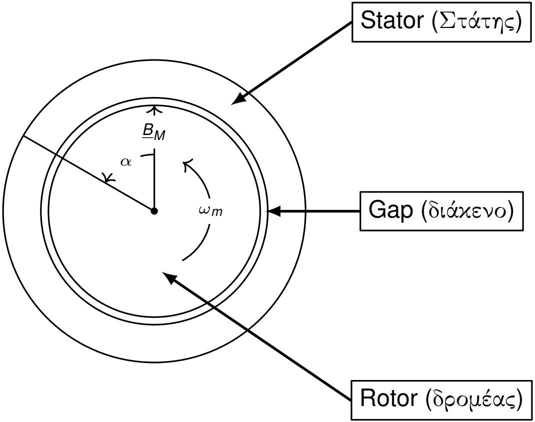
- In **all** three phase AC machines, the stator is fed with AC voltages, leading to a rotating magnetic field.
- In **synchronous (σύγχρονες)** machines, the rotor is fed with a DC current leading to a constant magnetic field.
  $\rightarrow$ As the constant magnetic field of the rotor tries to align to the rotating magnetic field of the stator, the rotor will rotate at constant *synchronous* speed (defined by the electrical frequency and the number of poles).
- In **induction (επαγωγής)** machines, the rotor is short-circuited (with a resistance) and alternating currents are induced by the stator field.
  $\rightarrow$ Think of it like a three-phase transformer: the AC currents in the stator (primary) generate a magnetic field that induces AC currents in the rotor (secondary).
  $\rightarrow$ The rotor does not rotate *synchronously* but it 'slips', meaning it operates at a different frequency than the stator.

2 Three-phase machine induced voltage

If we start rotating a rotor at a speed ω_m in a P-pole machine, the magnetic field observed at a constant location with angle α on the stator is now:

$$\boxed{B = B_M \cos(\omega t - \alpha)}$$

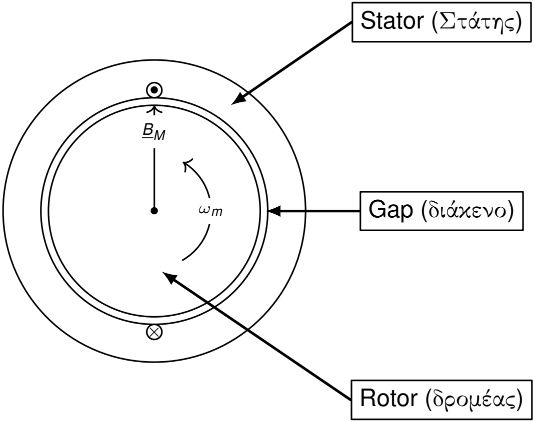
if we measure α from the direction of the peak flux density B_M and $\omega = \frac{P}{2}\omega_m$.



✍ Beware that the field in the rotor is generated by a DC current, thus it has a **constant** direction. The field fluctuation is caused by the mechanical rotation of rotor. This is not the same in the stator field we studied previously where the field was fluctuating due to the sinusoidal currents in the three-phase windings.

2 Three-phase machine induced voltage

We place one stator winding as shown below at the point of peak flux density ($\alpha = 0$):



The magnetic field generated by the rotor B_M is seen by the stator winding as a varying field given by $B = B_M \cos(\omega t)$.

2 Three-phase machine induced voltage

Due to the rotating field, there is an induced voltage on the stator winding given by Faraday law:

$$e = -\frac{d\lambda}{dt}$$

with λ the flux linkage given by $\lambda = N_c\phi = N_c\Phi_M\cos(\omega t)$ (N_c the number of winding turns on the stator). Thus, the induced voltage is given as:

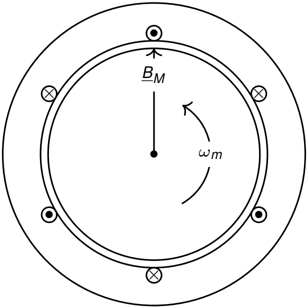
$$\boxed{e = -N_c\Phi_M\frac{d(\cos(\omega t))}{dt} = N_c\Phi_M\omega\sin(\omega t)}$$

In the particular case where the stator winding is rectangular with radius r and length ℓ, then the area is $A = 2r\ell$ and $\Phi_M = AB = 2r\ell B_M\cos(\omega t)$. Thus:

$$e = -2r\ell N_cB_M\frac{d(\cos(\omega t))}{dt} = 2r\ell N_cB_M\omega\sin(\omega t)$$

2 Three-phase machine induced voltage

Following the same analysis for three windings spaced 120° apart:



Gives (in Volt):

$$e_{aa'} = N_c \Phi_M \omega \sin(\omega t)$$
$$e_{bb'} = N_c \Phi_M \omega \sin(\omega t - 120^\circ)$$
$$e_{cc'} = N_c \Phi_M \omega \sin(\omega t - 240^\circ)$$

2 Three-phase machine induced voltage

The peak voltage at each phase is:

$$E_{max} = N_c \Phi_M \omega = N_c \Phi_M 2\pi f$$

with the RMS voltage:

$$\boxed{E_{RMS} = \frac{N_c \Phi_M 2\pi f}{\sqrt{2}} = \sqrt{2} N_c \Phi_M \pi f = 4.44 N_c \Phi_M f}$$

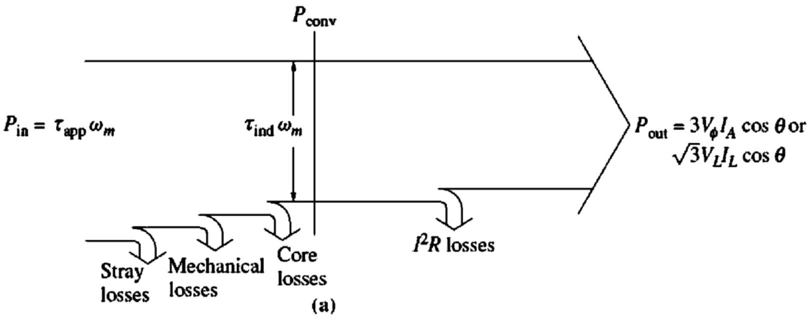
- If the generator is connected in Y, then it's voltage is $\sqrt{3}E_{RMS}$.
- If the generator is connected in Delta, then it's voltage is E_{RMS}.

In phasor representation:

$$\underline{E}_A = E_{RMS}\angle 0^\circ$$
$$\underline{E}_B = E_{RMS}\angle -120^\circ$$
$$\underline{E}_C = E_{RMS}\angle -240^\circ$$

3 Outline

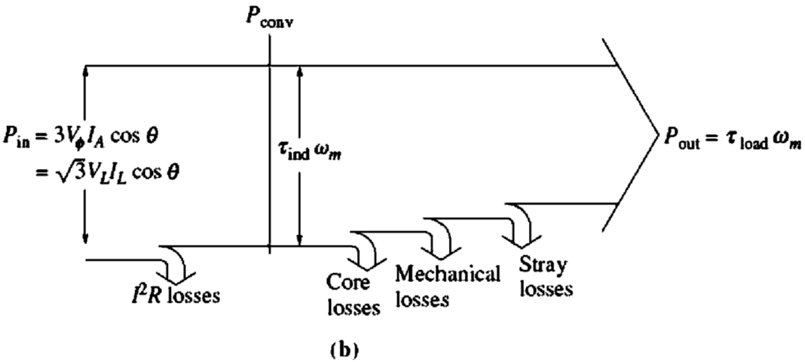
3 Power flows: Generator operation



- **Electrical losses:** $P = 3I^2R$
- **Core losses:** Losses in magnetic core (hysterisis, eddy currents, etc.)
- **Mechanical losses:** Friction and windage
- **Stray losses:** Everything not included above ($\approx 1\%$)

Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

3 Power flows: Motor operation



Chapman, S.J. (2005). Electric machinery fundamentals (4e). McGraw-Hill.

3 Swing equation

The equation governing the rotor motion is called the *swing equation*:

$$J\frac{d^2\theta_m}{dt^2} = J\frac{d\omega_m}{dt} = T_a = T_m - T_e \quad \text{N-m}$$

where:

- J is the total moment of inertia of the rotor mass in $kg - m^2$
- θ_m is the angular position of the rotor with respect to a stationary axis in (rad)
- $\omega_m = \frac{d\theta_m}{dt}$ is the angular speed of the rotor with respect to a stationary axis in (rad/s)
- t is time in seconds (s)
- T_m is the mechanical torque supplied by the prime mover in N-m
- T_e is the electrical torque output of the alternator in N-m
- T_a is the net accelerating torque, in N-m

3 Swing equation

Multiplying both sides by ω_m, the can rewrite the equations as:

$$J\omega_m \frac{d^2\theta_m}{dt^2} = J\omega_m \frac{d\omega_m}{dt} = P_a = P_m - P_e \quad \text{W}$$

where P_a, P_m and P_e are the net, mechanical and electrical powers, respectively.

A useful representation is by introducing the inertia constant of the machine:

$$H = \frac{\text{stored kinetic energy in mega joules at synchronous speed}}{\text{machine rating in MVA}} = \frac{J\omega_s^2}{2S_{rated}} \text{ MJ/MVA}$$

where S_{rated} is the three-phase power rating of the machine in MVA.

3 Efficiency

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The power efficiency of a machine is:

$$\eta = \frac{P_{out}}{P_{in}} \times 100\%$$

The voltage regulation is a measure of the ability of a generator to keep constant voltage at its terminals as load varies:

$$VR = \frac{V_{no-load} - V_{full-load}}{V_{full-load}} \times 100\%$$

A small VR is ”better” as the voltage is more constant.

The speed regulation is a measure of the ability of a motor to keep constant speed as load varies:

$$SR = \frac{n_{no-load} - n_{full-load}}{n_{full-load}} \times 100\%$$

$$SR = \frac{\omega_{no-load} - \omega_{full-load}}{\omega_{full-load}} \times 100\%$$

A small *SR* is ”better” as the speed is more constant.

3 Summary

- Electromechanical energy conversion is achieved through the interaction of the magnetic fields in the stator and the rotor.
- Forces and torques develop to align the magnetic fields, dictated by the Lorentz law.
- In a generator, the magnetic field of the rotor induces voltages to the stator windings.
- In a motor, the rotating magnetic field of the stator induces a torque on the rotor and on the load.