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EEN442 - Power Systems II (Συστήματα Ισχύος II)

Part 1: Revision of power engineering fundamentals

https://sps.cut.ac.cy/courses/een442/

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Overview

This course extends into the field of electric power systems based on the course EEN320. It focuses on the use of computational methods for the analysis of electric power systems in steady-state as well as during symmetrical and asymmetrical faults. It proceeds to provide the fundamental understanding of protection devices operation and selection.

Learning outcomes

On completion of this course, students should be able to:

- Understand the fundamental computational methods to analyze the steady-state and under-fault operation of electric power systems;
- Understand and analyze the basic power transfer limits in electric power systems; and,
- Understand the fundamentals of protection devices and protection selection.

Prerequisites

The following course knowledge are prerequisites for this course:

1. Engineering mathematics (advanced mathematics I-III, linear algebra)
2. Electric circuit analysis I-II
3. Power systems I (EEN320).

Syllabus

The course consists of these parts:

1. Revision of power engineering fundamentals (per-unit, models of lines, transformers, generators and loads)
2. Fundamentals of power system operation (Surge impedance loading, lossless line, efficiency, loadability, maximum power over line, P-δ and P-V characteristics, Ferranti effect)
3. Power flow analysis (nodal admittance matrix, NR-method, fast decoupled, DC power flow)
4. Unbalanced operation (symmetrical components)
5. Fault analysis (models of line, transformer, and generator during fault, solid faults)
6. Protection fundamentals (fundamentals, structure, overcurrent, distance, differential, digital relays)

Bibliography

1. A. Gómez-Expósito, A. J. Conejo, and C. A. Canizares, Electric Energy Systems Analysis and Operation, 2nd edition, CRC Press, 2018.
2. D. Glover, M. S. Sarma and T. Overbye, Power System Analysis & Design, 6th edition, Cengage Learning, 2017.
3. Nasser Tleis, Power Systems Modelling and Fault Analysis : Theory and Practice, 2nd edition, Academic Press, 2019.
4. Ν. Βοβός, Γ. Γιαννακόπουλος, "Ανάλυση Συστημάτων Ηλεκτρικής Ενέργειας", Β' έκδοση, εκδόσεις ΖΗΤΗ, 2019

Delivery

1. Theory delivered through lectures (in class $\approx$ 24 hours)
2. Practical examples (in class $\approx$ 8 hours)
3. Software laboratory work (in class $\approx$ 10 hours)
   a. Use of DigSILENT Power Factory software
   b. Surge Impedance and No-load conditions, critical loading, reactive compensation, N-1 secure expansion planning.
4. Hardware laboratory work (in class $\approx$ 12 hours)
   The laboratory this year will take place abroad at HDA (Germany) from 28.10.2024-01.11.2024.
   The topics will cover:
   a. Electric Vehicles
   b. PV inverters
   c. SCADA systems

Assessment

1. Mid-term exam (20%)
2. Software laboratory exam (20%)
3. Hardware laboratory report (20%)
4. Final written exam (40%)

Questions?

Learning objectives

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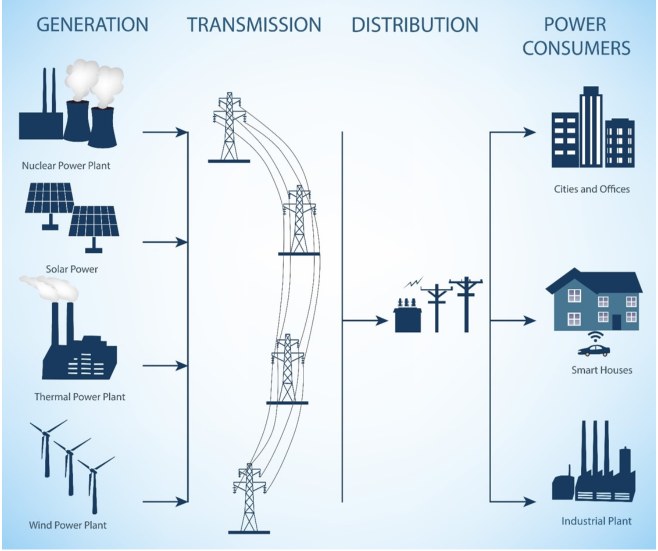
After this part of the lecture and additional reading, you should be able to ...

1. ...describe the basic operation of power systems;
2. ...use the per-unit system to perform analysis of power systems;
3. ...describe and use the models used for the basic power system components (line/transformer/generator/load models).

1 Outline

1.1 Electric power system overview



1.1 Transmission and distribution of electric energy

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- Often, economic, geographic, environmental or technological reasons impede generation of *all* demand closed to load centres (cities, industrial sites)
- Therefore, large share of electric power is generated far away from load centres

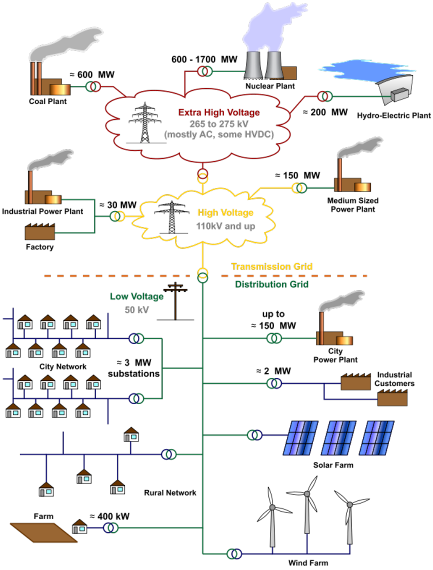
→ Need (electric) infrastructure to transport electricity from generators to loads

- This infrastructure is called a power network
- Fundamental components of a power network are
  - Conductors/power lines to transport electric current (overhead lines and cables)
  - Power transformers to modify voltage levels between different network parts
  - Protection equipment to disconnect (some) parts of a network in case of a failure

1.1 Voltage levels and network types

- Optimal economic interconnection of different generators / end-users / networks mainly depends on
  - Distance
  - Amount of power to be transmitted
- Consequently, most power systems worldwide consist of
  - **Transmission network**: *global* power network over large distances; works at high voltages
  - **Distribution network**: *local* electricity network to deliver power to end-users; works at medium and low voltage
  - Voltage usually transformed several times to lower values the closer to end-user
  - These voltage transformations are performed in **substations**
  - Above low voltage (LV) level, power transfer is usually three-phase

1.1 Electric power systems - Standard structure (1)



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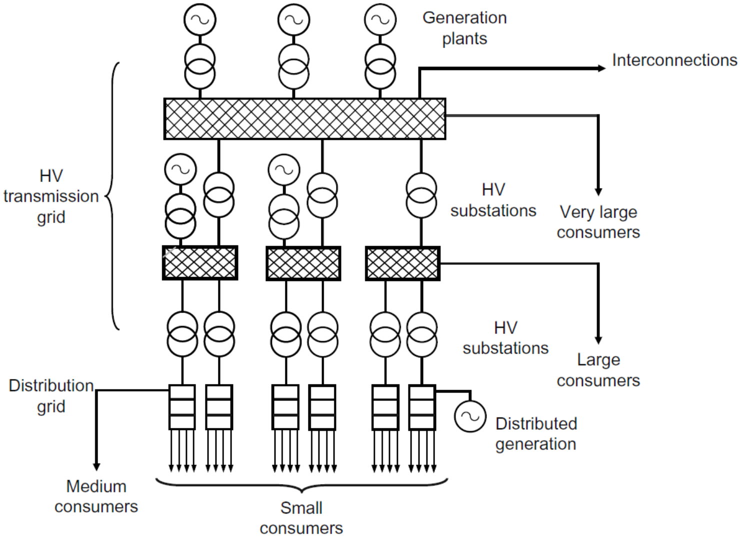
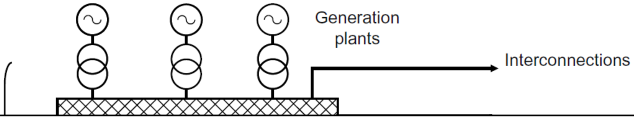
1.1 Electric power systems - Standard structure (2)



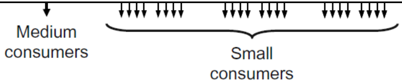
Source: J. Machowski et al, "Power system dynamics: stability and control", John Wiley & Sons, 2011

1.1 Electric power systems - Standard structure (2)



Characteristics

- Few, very large generators 1 - 1,000 MW (coal, nuclear, gas, hydro)
- Large generators connected to HV transmission system
- Role of HV transmission system: bulk power transport from generators to load centers (e.g. cities)
- Close to load centers, voltage transformation to lower levels in distribution network
- Unidirectional flow from HV to LV



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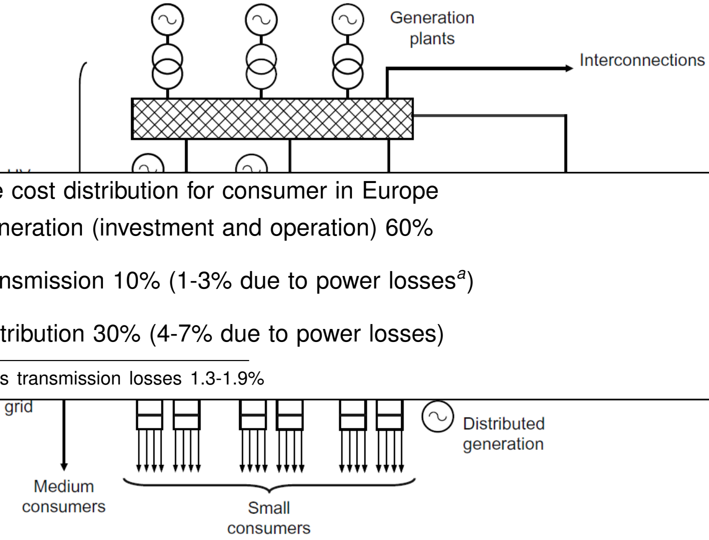
1.1 Electric power systems - Standard structure (2)



Average cost distribution for consumer in Europe

- Generation (investment and operation) 60%
- Transmission 10% (1-3% due to power losses[a])
- Distribution 30% (4-7% due to power losses)

[a] Cyprus transmission losses 1.3-1.9%

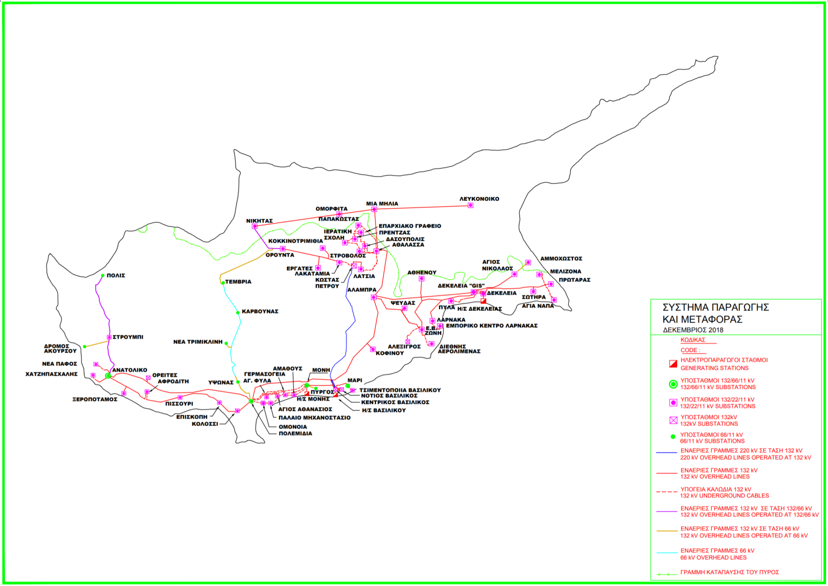
Source: J. Machowski et al, "Power system dynamics: stability and control", John Wiley & Sons, 2011

1.1 Voltage levels today

Country	high voltage (HV)	medium voltage (MV)	low voltage (LV)
UK	400 kV (275 kV)	132 kV - 11 kV	400 / 230 V
Germany	380 kV	110 - 10 kV	400 / 230 V
US	765 - 345 kV	230 - 4 kV	480 / 120 V
Nigeria	330 - 132 kV	33-10 kV	415/240 V
Cyprus	132 - 66 kV	22-11 kV	400/230 V

- The above voltage magnitudes refer to the line-to-line voltage V_{LL} of the corresponding three-phase system
- The line-to-ground voltage V_{LG} is given by $V_{LL} = \sqrt{3}V_{LG}$

1.1 Example of Cyprus



Source: https://www.dsm.org.cy/

1.2 Phasors in electrical power systems

- Sinusoidal waveforms can also be represented by *phasors* in the complex plane
- Phasors are very popular in electric power systems
- Main reasons: simplify visualisation and calculation of electrical networks
- This is very useful for analysis, design and operation of power systems

1.2 Definition of a phasor (φασιθέτης)

- Consider

$$x(t) = \hat{X}\cos(\omega t + \theta)$$

- Via Euler's Formula, we define the phasor corresponding to $x(t)$ as[1]

$$\underline{X} = \frac{\hat{X}}{\sqrt{2}}\left(\cos(\theta) + j\sin(\theta)\right) = \underbrace{X\left(\cos(\theta) + j\sin(\theta)\right)}_{\text{trigonometric form}} = \underbrace{Xe^{j\theta}}_{\text{exponential form}}$$

- Then

$$x(t) = \sqrt{2}\Re\{\underline{X}e^{j\omega t}\},$$

i.e., momentary value of $x(t)$ corresponds to real part of the phasor $\underline{X}$ rotating at angular speed ω

- Alternative common notation for a phasor

$$\underline{X} = Xe^{j\theta} = \underbrace{X\underline{/\theta}}_{\text{angular form}}$$

[1] Here j denotes the imaginary unit.

1.2 Voltages and currents as complex phasors

- Phasors of voltage and current

$$\underline{V} = V\left(\cos(\varphi_v) + j\sin(\varphi_v)\right) = Ve^{j\varphi_v} = V\underline{/\varphi_v}$$
$$\underline{I} = I\left(\cos(\varphi_i) + j\sin(\varphi_i)\right) = Ie^{j\varphi_i} = I\underline{/\varphi_i}$$
$$\varphi = \varphi_v - \varphi_i$$

- Note: as we have assumed stationary conditions, it suffices to use $\underline{X}$ to describe $x(t)$ for network calculations

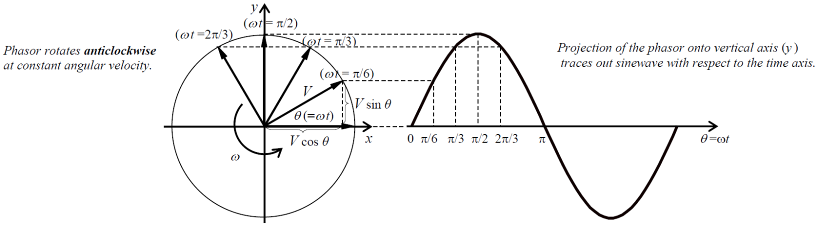
  Why? Because the term $e^{j\omega t}$ cancels out, whenever multiplying two complex quantities

$$\left(\underline{V}e^{j\omega t}\right)\left(\underline{I}e^{j\omega t}\right)^* = \underline{V}\,\underline{I}^* e^{j\omega t}e^{-j\omega t} = \underline{V}\,\underline{I}^*,$$

  where the operator * denotes complex conjugation

1.2 Visualization of a phasor

$$\underline{V} = Ve^{j\theta} = \underbrace{V\underline{/\theta}}_{\text{angular form}}$$



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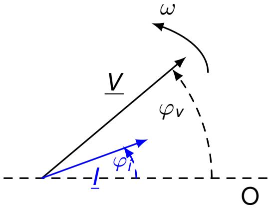
- starting from the origin $0 + j0$
- projection on the real axis is $\frac{1}{\sqrt{2}}v(t)$
- the phasor is the position at $t = 0$ of the rotating vector

1.2 Phasor diagrams

- Phasor diagrams (φασικά διαγράμματα): A graphical representation of the phasors



1.3 Complex apparent power in single-phase systems

- Now, we can introduce a third important quantity in power systems - the complex apparent power

$$\underline{S} = \underline{V}\,\underline{I}^* = VIe^{j(\varphi_v - \varphi_i)} = VIe^{j\varphi} = VI(\cos(\varphi) + j\sin(\varphi))$$

- Remember that $\varphi = \varphi_v - \varphi_i$ is called the power factor angle and it's connected to power factor as $PF = cos\varphi$

- The absolute value of the complex apparent power is called apparent power S

$$S = |\underline{S}| = VI$$

- The unit of $\underline{S}$ and S is Volt-Ampere [VA]

- Apparent power used to dimension equipment

$$S = VI \quad \Rightarrow \quad S = P \quad \text{if} \quad \varphi = 0$$

1.3 Relation between $\underline{S}$, P and Q

- Active power P corresponds to real part of $\underline{S}$

$$P = \Re\{\underline{S}\} = VI\cos(\varphi)$$

- Reactive power Q corresponds to imaginary part of $\underline{S}$

$$Q = \Im\{\underline{S}\} = VI\sin(\varphi)$$

- Hence

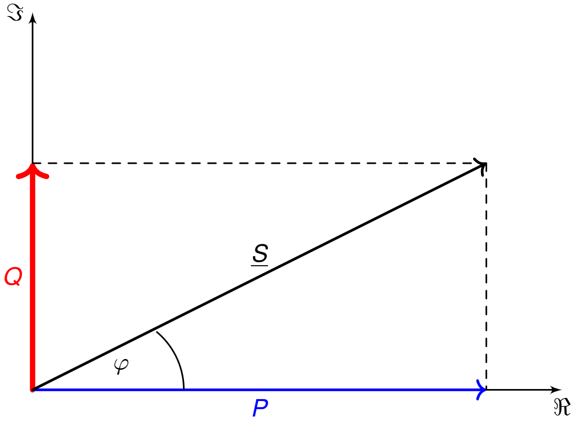
$$\underline{S} = P + jQ$$

and

$$S = |\underline{S}| = \sqrt{P^2 + Q^2}$$

$p(t)$, P	Watt	W	kW, MW
Q	Var	(VAr, Var, var)	kvar, Mvar
S	Volt-Ampere	VA	kVA, MVA

1.3 Power triangle in the complex plane



Power factor

$$\cos(\varphi) = \frac{P}{|\underline{S}|} = \frac{P}{\sqrt{P^2 + Q^2}}$$

1.3 Complex three-phase AC power

- Under balanced conditions, the complex three-phase AC power is defined as

$$
\begin{aligned}
\underline{S}_{3\phi} &= \underline{V}_a \underline{I}_a^* + \underline{V}_b \underline{I}_b^* + \underline{V}_c \underline{I}_c^* \\
&= \underline{V}_a \underline{I}_a^* + \underline{V}_a e^{-j\frac{2\pi}{3}} \underline{I}_a^* e^{j\frac{2\pi}{3}} + \underline{V}_a e^{-j\frac{4\pi}{3}} \underline{I}_a^* e^{j\frac{4\pi}{3}} \\
&= 3\underline{V}_a \underline{I}_a^* \\
&= 3VIe^{j\varphi} \\
&= 3VI\cos(\varphi) + j3VI\sin(\varphi) \\
&= 3P_a + j3Q_a \text{ [VA]}
\end{aligned}
$$

- Three-phase active power: $P_{3\phi} = \Re\{\underline{S}_{3\phi}\} = 3VI\cos(\varphi) = 3P_a$ [W]
- Three-phase reactive power: $Q_{3\phi} = \Im\{\underline{S}_{3\phi}\} = 3VI\sin(\varphi) = 3Q_a$ [Var]

Under stationary and balanced conditions, total three-phase active power transmitted over a three-phase element is constant!

1.3 Complex three-phase AC power using line voltages

- Complex three-phase power

$$\underline{S}_{3\phi} = 3\underline{V}_{LN}\underline{I}_L^* = 3VIe^{j\varphi} = 3VI\cos(\varphi) + j3VI\sin(\varphi)$$

- With $\sqrt{3}\underline{V}_{LN} = \underline{V}_{LL}$ and $|\sqrt{3}\underline{V}_{LN}| = |\underline{V}_{LL}| = \sqrt{3}V = U$

$$\underline{S}_{3\phi} = \sqrt{3}\underline{V}_{LL}\underline{I}_L^* = \sqrt{3}UI\cos(\varphi) + j\sqrt{3}UI\sin(\varphi)$$

- These formulae are “hybrid” in so far as:
  - V_{LL} is the effective value of the *line voltage*
  - φ is the phase angle between the line current and the *phase-to-neutral voltage*.
- Three-phase active power: $P_{3\phi} = \Re\{\underline{S}_{3\phi}\} = \sqrt{3}UI\cos(\varphi)$
- Three-phase reactive power: $Q_{3\phi} = \Im\{\underline{S}_{3\phi}\} = \sqrt{3}UI\sin(\varphi)$

1.4 Principle of ”per unit” system

- Usual representation of physical quantities as product of numerical value and physical unit, e.g.

$$V = 400 \text{ kV}$$

- Alternative: representation of the quantity *relative* to another (base) quantity

$$\boxed{\text{value of quantity in pu} = \frac{\text{value of quantity in physical unit}}{\text{value of corresponding ”base” in same unit}}}$$

- Division by ”base” eliminates physical unit

→ per-unit (pu) system

- Example: base value for voltage $V_{\text{base}} = 400$ kV

$$\frac{V}{V_{\text{base}}} = \frac{400 \text{ kV}}{400 \text{ kV}} = 1 \text{ pu}$$

1.4 Per unit quantities - Summary

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1. Out of the three base quantities (S_B, V_B, I_B) choose two. Usually: S_B and V_B

   S_B can be either single- or three-phase power, where

   $$\boxed{S_{B3\phi} = 3S_{B1\phi}}$$

2. Other values obtained via electrical laws

	Single-phase	Three-phase
Base current	$I_B = \frac{S_{B1\phi}}{V_B}$	$I_B = \frac{S_{B3\phi}}{3V_B} = \frac{S_{B3\phi}}{\sqrt{3}U_B}$
Base impedance	$Z_B = \frac{V_B^2}{S_{B1\phi}}$	$Z_B = \frac{3V_B^2}{S_{B3\phi}} = \frac{U_B^2}{S_{B3\phi}}$
Base admittance	$Y_B = \frac{1}{Z_B}$	$Y_B = \frac{1}{Z_B}$

- V_B and I_B are always RMS values per phase!
- $U_B = \sqrt{3}V_B$ is the line voltage.
- In non-stationary conditions usually frequency and/or time are also normalised

1.4 Conversion between different per unit systems

- In practice, it is often necessary to convert values from one per unit system to another one
- Example: machine parameters are given in per unit values with respect to machine rating and we want to convert them into per unit values with respect to base values of power system to which machine is connected
- This can be done as follows

  Per unit value wrt first base: $x_1 = \frac{X}{X_{B,1}}$

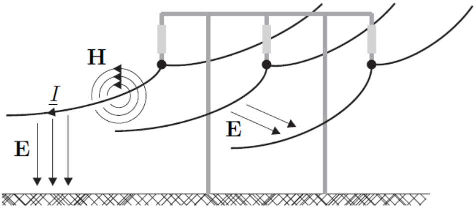
  Per unit value wrt second base: $x_2 = \frac{X}{X_{B,2}}$

  Hence: $X = x_1 X_{B,1} = x_2 X_{B,2}$

$\rightarrow$ Conversion from base 1 to base 2:

$$\boxed{x_2 = x_1 \frac{X_{B,1}}{X_{B,2}}}$$

Magnetic and electric fields of conducting power line



E. . . electric field

H. . . magnetic field

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- Each power line has characteristic line parameters
- Parameters dependent on line geometry and material
- Parameters often indicated in [unit]/km and by giving the line length ℓ

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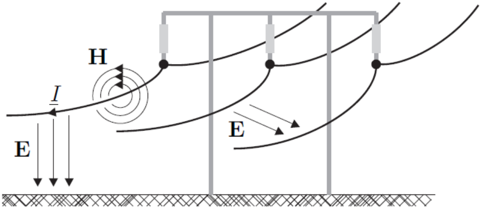
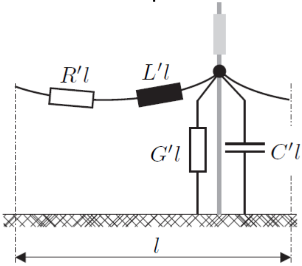
Magnetic and electric fields of conducting power line



Power line model with concentrated parameters



- Line resistance R' [Ω/km] $\leftrightarrow$ Ohmic resistance of conductor
- Line inductance L' [H/km] $\leftrightarrow$ Magnetic field of conductor
- Capacitance C' [F/km] $\leftrightarrow$ Electric field of conductor
- Shunt conductance G' [S/km] $\leftrightarrow$ Leakage currents at insulators

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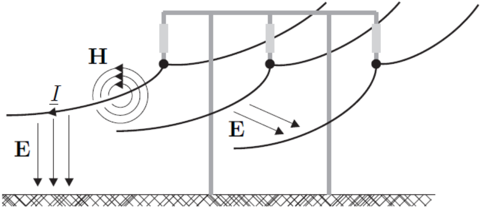
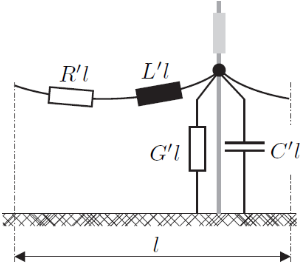
Magnetic and electric fields of conducting power line



Power line model with concentrated parameters



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- For performing circuit analysis involving power lines (e.g. to determine the network conditions or design) we need to know the concentrated parameters of the lines
- Usually, concentrated parameters indicated by manufacturer

Please see the course book for a detailed derivation.

2.1 Resistance - Calculation

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- For steel-reinforced aluminium conductors (ACSR), AC resistance is approximately same as DC resistance
- Reason: Skin-effect $\rightarrow$ reduced AC current in steel strands $\rightarrow$ increase in AC resistance by skin-effect comparable to higher DC current in steel strands
- Conductor losses result in heat dissipation $\rightarrow$ maximum conductor current limited, as long-term high temperatures ($> 80^\circ$) decrease mechanical strength of conductor material $\rightarrow$ line sags
- Line resistance operating at temperature of ϑ° can be calculated via

$$R' = R'_{20}(1 + \alpha(\vartheta - 20^\circ\text{C}))\ [\text{R/m}]$$

$$R'_{20} = \frac{\rho_{20}}{A} \text{ resistance of conductor at } 20^\circ\text{C}$$

$\rho_{20}\ldots$ specific resistance of of conductor material at 20°C

$A\ldots$ effective conductor area

- For practical conductors, resistance values obtained via measurements

2.1 Conductance

- Also, losses due to insulator leakage currents and corona
- Corona: high value of electric field strength at conductor surface causes air to become electrically ionised and to conduct
- Corona losses dependent on meteorological conditions (rain; humidity) and conductor surface irregularities
- For overhead lines, conductance G' can only be estimated from measurements, while it can be determined experimentally for cables
- Usually, conductance is very small and therefore most often neglected in power system studies

2.2 Cables vs. overhead lines

- Cables mostly used at low voltage levels ($<$110 kV)
- Often installed underground
- Physical characteristics of cables fundamentally different from overhead transmission lines (OHLs)!
- Main reasons:
  - Distance between conductors as well as between conductors and earth much smaller in cables than in OHLs
  - Conductors in cables typically surrounded by other metallic materials, e.g. skin
  - Insulation material of OHLs is air, while in cables materials such as paper, oil or SF_6 are used
- Consequences:
  - Inductance of OHLs usually higher as that of cables
  - Capacitance of cables usually much higher as that of OHLs

2.2 Cables vs. overhead lines - Typical parameters

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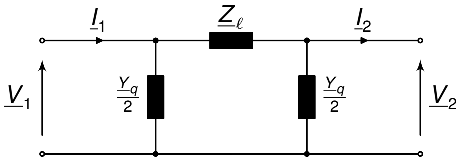
- Typical values for parameters of OHLs at 50 Hz

Rated voltage in kV	230	345	500	765
R' [Ω/km]	0.050	0.037	0.028	0.012
$X'_L = \omega L'$ [Ω/km]	0.407	0.306	0.271	0.274
$Y'_C = \omega C'$ [μS/km]	2.764	3.765	4.333	4.148

- Typical values for parameters of cables at 50 Hz

Rated voltage in kV	115	230	500
R' [Ω/km]	0.059	0.028	0.013
$X'_L = \omega L'$ [Ω/km]	0.252	0.282	0.205
$Y'_C = \omega C'$ [μS/km]	192.0	204.7	80.4

2.3 Π-equivalent circuit of homogeneous power line – Full model



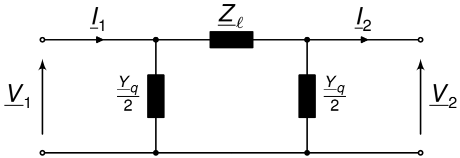
where:

$$\underline{Z}_\ell = \underline{Z}_W \sinh(\underline{\gamma}\ell)$$

$$\frac{\underline{Y}_q}{2} = \frac{\cosh(\underline{\gamma}\ell) - 1}{\underline{Z}_W \sinh(\underline{\gamma}\ell)} = \frac{1}{\underline{Z}_W} \tanh\left(\frac{\underline{\gamma}\ell}{2}\right)$$

$$\boxed{\begin{aligned} \underline{\gamma} &= \sqrt{(R' + j\omega L')(G' + j\omega C')} \\ \underline{Z}_w &= \sqrt{\frac{R' + j\omega L'}{G' + j\omega C'}} \end{aligned}}$$

These parameters correspond to exact relations between currents and voltages according to wave equation for $x = 0$ and $x = \ell$



- For $|\underline{\gamma}\ell| \ll 1$, the expressions for $\underline{Z}_\ell$ and $\underline{Y}_q$ can be simplified

$$\underline{Z}_\ell = \underline{Z}_W \sinh(\underline{\gamma}\ell) \approx \underline{Z}_W \underline{\gamma}\ell = \underline{Z}'\ell$$

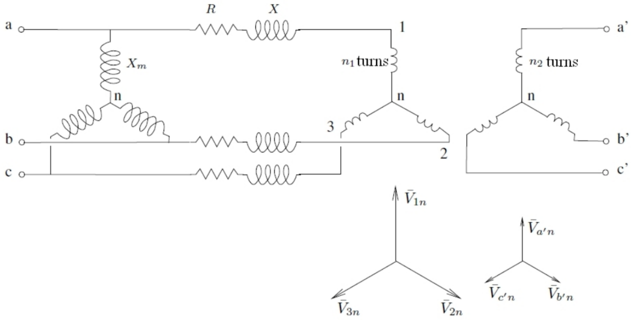
$$\frac{\underline{Y}_q}{2} = \frac{1}{\underline{Z}_W} \tanh\left(\frac{(\underline{\gamma}\ell)}{2}\right) \approx \frac{1}{\underline{Z}_W}\frac{\underline{\gamma}\ell}{2} = \frac{\underline{Y}'\ell}{2}$$

→ *Concentrated* elements $\underline{Z}_\ell$ and $\underline{Y}_q$ can be computed from *distributed* parameters R', L', G' and C' if $|\underline{\gamma}\ell| \ll 1$

$$\boxed{\begin{aligned} \underline{Z}_\ell &= \underline{Z}'\ell = (R' + jX')\ell \\ \frac{\underline{Y}_q}{2} &= \frac{\underline{Y}'}{2}\ell = \frac{(G' + jB')}{2}\ell \end{aligned}}$$

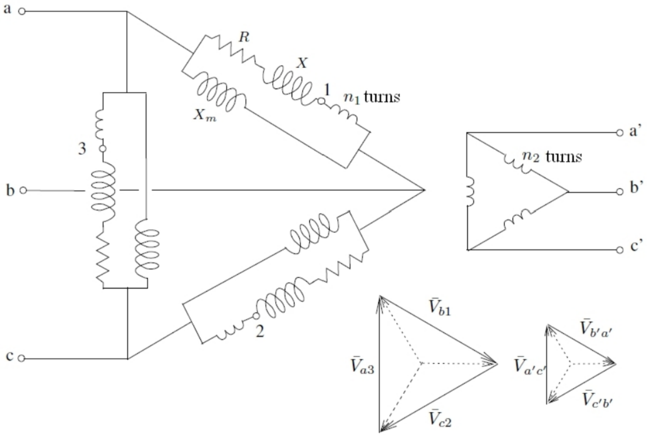
Four main different configuration possibilities:Y-Y, Y-Delta, Delta-Y, Delta-Delta

1. Y-Y-configuration



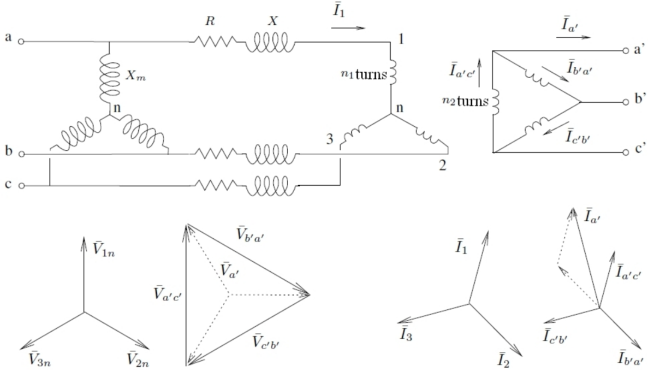
- Preferred at very high voltage level since voltage across each coil is $\sqrt{3}$ lower
- Possibility to connect neutral to ground (safety protection)

2. Delta-Delta-configuration



- Preferred for high currents, as currents in phases $\sqrt{3}$ lower
- Also used to eliminate harmonics

3. Y-Delta-configuration



- Frequent transformer configuration connecting generator to grid
- On high-voltage side, neutral point is grounded (for protection)

3 Three-phase transformer - Classification

- Standardized abbreviation of I.E.C. (International Electrotechnical Commission)
- Classification of transformer configuration consists of 2 letters and 1 integer
- First letter: Configuration of high-voltage side; upper-case letter used (**Y** or **D**)
- Second letter: Configuration of low-voltage side; lower-case letter used (**y** or **d**)
- Integer: Phase shift between voltages at primary and secondary winding of the same phase as multiple of 30° ($\pi/6$) assuming the transformer is ideal
- Additionally in Y-connection: **n** after **Y** or **y** to indicate that neutral is grounded

3.1 Three-phase transformer - Transformation ratio (1)

- In balanced operation, in principle can use single-phase equivalent circuit for analysis
- But if analysing Yd or Dy connections, need to consider
  1. Not same voltages on primary and secondary side: one has phase voltages the other line voltages

  $\rightarrow$ Transformation ratio also affected!

  2. Phase and line voltages also differ in phase

  $\rightarrow$ Additional phase displacement Phase displacement is integer multiple of $\pi/6 = 30^\circ$

3.1 Three-phase transformer - Transformation ratio (2)

- Impact on amplitude considered by introducing additional scalar k
  - Dy-configuration: $k = \frac{1}{\sqrt{3}}$
  - Yd-configuration: $k = \sqrt{3}$
  - Yy or Dd: $k = 1$
- Impact on phase displacement considered by introducing additional phase shift element

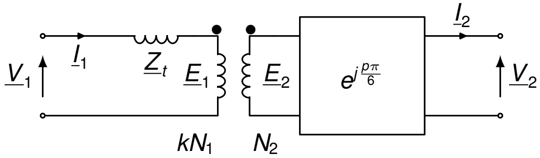
$$e^{j\frac{p\pi}{6}},$$

where $p = \{0, 1 \ldots, 11\}$ is an integer

$\rightarrow$ Complex transformation ratio

$$\boxed{\underline{c} = k\frac{N_1}{N_2}e^{j\frac{p\pi}{6}}}$$

- Note: $|e^{j\frac{p\pi}{6}}| \rightarrow$ amplitude of transformation not influenced by p



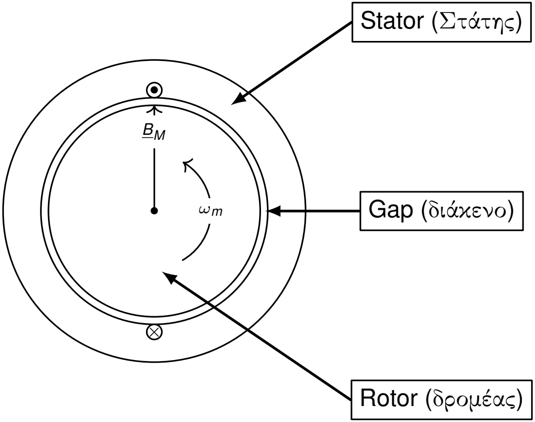
- Combine single-phase transformer model with complex transformation ratio
- Then

$$\underline{E}_1 = \underline{c}\,\underline{V}_2$$
$$\underline{I}_2 = \underline{c}^*\,\underline{I}_1$$

1. Basics
2. Transmission lines
3. Power transformers
4. **Rotating machines**
   - Synchronous machine
   - Induction machine
5. Electric loads

4.1 Three-phase machine induced voltage

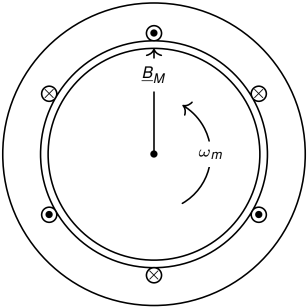
We place one stator winding as shown below at the point of peak flux density ($\alpha = 0$):



The magnetic field generated by the rotor B_M is seen by the stator winding as a varying field given by $B = B_M \cos(\omega t)$.

4.1 Three-phase machine induced voltage

Following the same analysis for three windings spaced 120° apart:



Gives (in Volt):

$$e_{aa'} = N_c \Phi_M \omega \sin(\omega t)$$
$$e_{bb'} = N_c \Phi_M \omega \sin(\omega t - 120^\circ)$$
$$e_{cc'} = N_c \Phi_M \omega \sin(\omega t - 240^\circ)$$

4.1 Three-phase machine induced voltage

The peak voltage at each phase is:

$$E_{max} = N_c \Phi_M \omega = N_c \Phi_M 2\pi f$$

with the RMS voltage:

$$\boxed{E_{RMS} = \frac{N_c \Phi_M 2\pi f}{\sqrt{2}} = \sqrt{2} N_c \Phi_M \pi f = 4.44 N_c \Phi_M f}$$

- If the generator is connected in Y, then it's voltage is $\sqrt{3}E_{RMS}$.
- If the generator is connected in Delta, then it's voltage is E_{RMS}.

In phase representation:

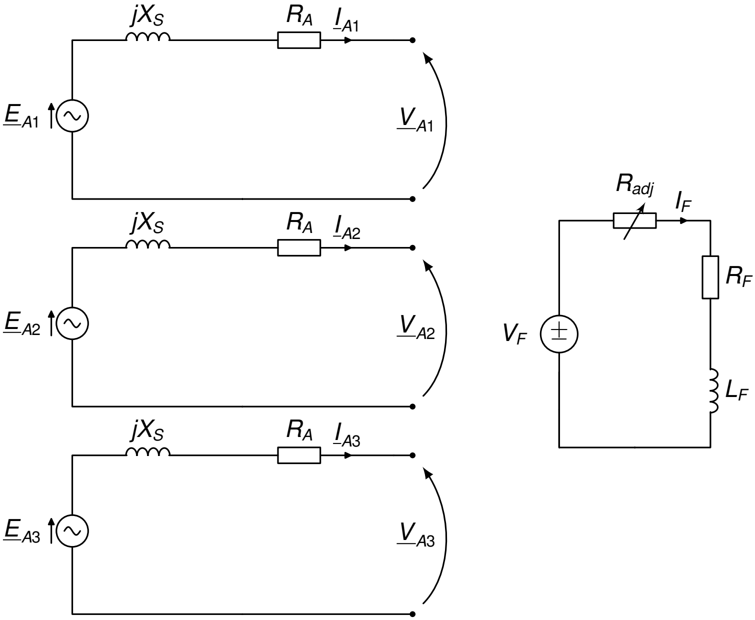
$$\underline{E}_A = E_{RMS}\angle 0^\circ$$
$$\underline{E}_B = E_{RMS}\angle -120^\circ$$
$$\underline{E}_C = E_{RMS}\angle -240^\circ$$

The (simplified) equivalent model is given:

$$\boxed{\underline{V}_A = \underline{E}_A - jX\underline{I}_A - jX_A\underline{I}_A - R_A\underline{I}_A = \underline{E}_A - jX_S\underline{I}_A - R_A\underline{I}_A}$$

with $X_S = X + X_A$ the synchronous reactance of the generator.

4.1 (Simplified) equivalent circuit of synchronous generator



The output power is:

$$P_{out} = 3V_A I_A \cos(\theta) \quad Q_{out} = 3V_A I_A \sin(\theta)$$

where θ is the angle between $\underline{V}_A$ and $\underline{I}_A$.

If we ignore the resistance R_A (since $R_A << X_S$), we can use the power flow equations over a reactance to get the generator power output and torque:

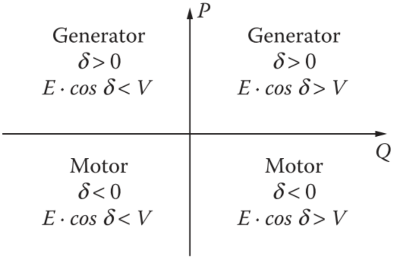
$$\boxed{P = \frac{3V_A E_A}{X_S}\sin(\delta)} \quad \boxed{\tau = \frac{3V_A E_A}{\omega_m X_S}\sin(\delta)} \quad \boxed{Q = \frac{3V_A E_A}{X_S}\cos(\delta) - \frac{3V_A^2}{X_S}}$$

with δ the angle between $\underline{E}_A$ and $\underline{V}_A$, also called *torque angle*.

Q1: What is the maximum power of the generator, if we keep E_A and V_A constant?
Q2: Can you rewrite the equations with line voltages instead of phase voltages?

4.1 Synchronous generator working conditions

The synchronous generator operates as generator or motor:



The following limits are imposed:

- Mechanical power limits $P_{m,max}$ and $P_{m,min}$
- Stator thermal limit I_{max}
- Maximum internal voltage limit E_{max}
- Stability limit δ_{max}

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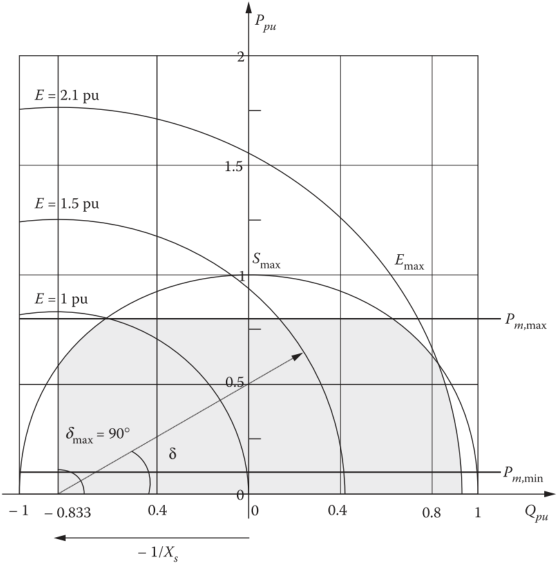
Example 2.6:



4.1 Swing equation lossless machine

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The equation governing the rotor motion is called the *swing equation*:

$$J\frac{d^2\theta_m}{dt^2} = J\frac{d\omega_m}{dt} = T_a = T_m - T_e \quad \text{[N-m]}$$

where:

- J is the total moment of inertia of the rotor mass in $kg - m^2$
- θ_m is the angular position of the rotor with respect to a stationary axis in (rad)
- $\omega_m = \frac{d\theta_m}{dt}$ is the angular speed of the rotor with respect to a stationary axis in (rad/s)
- t is time in seconds (s)
- T_m is the mechanical torque supplied by the prime mover in N-m
- T_e is the electrical torque output of the alternator in N-m
- T_a is the net accelerating torque, in N-m

Multiplying both sides by ω_m, the can rewrite the equations as:

$$M\frac{d^2\theta_m}{dt^2} = M\frac{d\omega_m}{dt} = P_a = P_m - P_e \quad [\text{W}]$$

where P_a, P_m and P_e are the net, mechanical and electrical powers, respectively. $M = J\omega_m$ is the angular momentum of the rotor.

A useful representation is by introducing the inertia constant of the machine:

$$H = \frac{\text{stored kinetic energy in mega joules at synchronous speed}}{\text{machine rating in MVA}} = \frac{J\omega_s^2}{2S_{rated}} \ \text{MJ/MVA}$$

where S_{rated} is the three-phase power rating of the machine in MVA.

4.1 Swing equation with losses

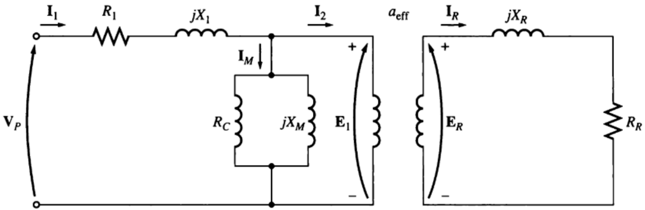
$$M\frac{d\omega_m}{dt} + D(\omega_m - \omega_s) = P_m - P_e - P_{losses} \quad [\mathrm{W}]$$

where P_{losses} are the electrical losses (stator) and D is the so-called damping constant of the machine representing mechanical rotational losses.

4.2 Induction machine

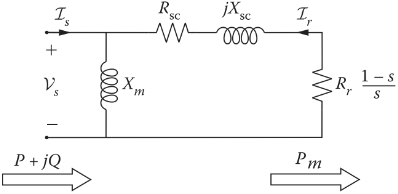
- The induction machine is an electrical machine in which the stator windings are fed through a three- phase voltage source, while the rotor windings are short circuited and are circulated by currents induced by the stator.

- In balanced steady-state conditions, the induction machine has an analog behavior to that of a transformer and hence a transformer model can be used to represent this machine.

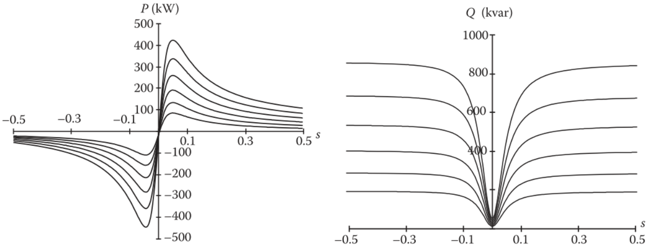


S. J. Chapman, Electric Machinery Fundamentals, 5th ed. McGraw-Hill, 2012.

4.2 Induction machine: Per-phase equivalent model



- P and Q: Active and reactive powers at the machine terminals.
- V_s and I_s: Voltage and current at the generator terminals (stator).
- $Z_{sc} = R_{sc} + jX_{sc}$: Short-circuit impedance comprising that of the stator plus that of the rotor referred to the stator.
- X_m: Per-phase reactance modeling the magnetization of the ferromagnetic core.
- s: Slip of the machine obtained as $s = (\omega_s - \omega)/\omega_s$, where ω_s is the synchronous speed in rad/s.
- P_m: Mechanical power exchanged by the machine with the outer world, which can be expressed as $P_m = I_r^2 R_r(1-s)/s$



- Special cases: $s = 1 \rightarrow$ locked-rotor, $s = 0 \rightarrow$ no-load
- Operating limits:
  - Stator thermal limit I_{max}
  - Dielectric insulation or maximum feeding voltage limit $V_{s,max}$
  - Stability or magnetizing limit (from curve)

A. Gomez-Exposito, A. J. Conejo, and C. A. Cañizares, Electric Energy Systems Analysis and Operation, 2018.

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One of the most well-known load models is the *exponential*:

$$\boxed{P(V) = P_0 \left(\frac{V}{V_0}\right)^{k_{PV}}} \qquad \boxed{Q(V) = Q_0 \left(\frac{V}{V_0}\right)^{k_{QV}}}$$

- $P(V), Q(V)$ are the active and reactive power consumed by the load at voltage V
- P_0, Q_0 are the active and reactive powers consumed at a voltage V_0
- k_{PV}, k_{QV} are load-voltage parameters depending on the type of load
  - $k_{PV} = k_{QV} = 2$: constant impedance load (Z)
  - $k_{PV} = k_{QV} = 1$: constant current load (I)
  - $k_{PV} = k_{QV} = 0$: constant power load (P)

5 Frequency-dependent load

Considering frequency sensitivity:

$$\boxed{P(V,f) = P(V)\left(1 + k_{Pf}\frac{f - f_0}{f_0}\right)} \qquad \boxed{Q(V,f) = Q(V)\left(1 + k_{Qf}\frac{f - f_0}{f_0}\right)}$$

- $P(V,f), Q(V,f)$ are the active and reactive power consumed by the load at voltage V and frequency f
- $P(V), Q(V)$ were given in previous slide
- f_0 is the nominal frequency (usually 50/60 Hz)
- k_{Pf}, k_{Qf} are load-frequency parameters depending on the type of load

Table 3.3 Typical load model parameters (IEEE, 1993)

Type of load	Power factor	k_{PV}	k_{QV}	k_{Pf}	k_{Qf}
Residential	0.87–0.99	0.9–1.7	2.4–3.1	0.7–1	−1.3 to −2.3
Commercial	0.85–0.9	0.5–0.8	2.4–2.5	1.2–1.7	−0.9 to −1.6
Industrial	0.8–0.9	0.1–1.8	0.6–2.2	−0.3–2.9	0.6–1.8

J. Machowski, J. Bialek, and J. Bumby, Power system dynamics: stability and control. JohnWiley and Sons, 2008.