EEN442 - Power Systems II (Συστήματα Ισχύος II)

Part 4: Unbalanced operation

https://sps.cut.ac.cy/courses/een442/

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This part's learning objectives

After this part of the lecture and additional reading, you should be able to . . .

1. . . . explain the use of symmetrical components to describe the unbalanced operation of three-phase power systems in steady-state;
2. . . . perform simple computations and system analysis with symmetrical components;
3. . . . explain the power balance in ABC and symmetrical components.

Outline

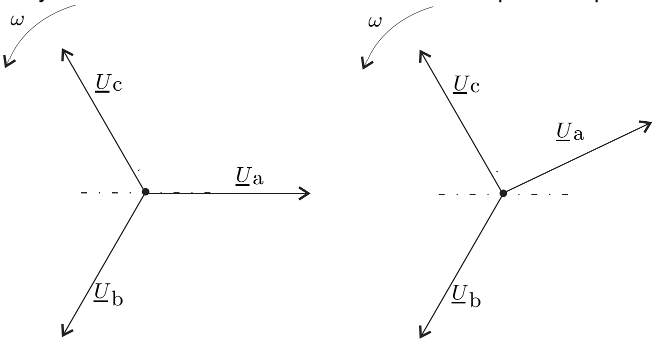
Unbalanced operation

Why would we have unbalanced operation?

- Unbalanced loads;
- unbalanced line parameters (e.g., untransposed and therefore, asymmetrical lines);
- unbalanced transformer parameters;
- ground faults or short circuits;

Unbalanced operation

Why don't we use the same methods as in the previous parts of this course?



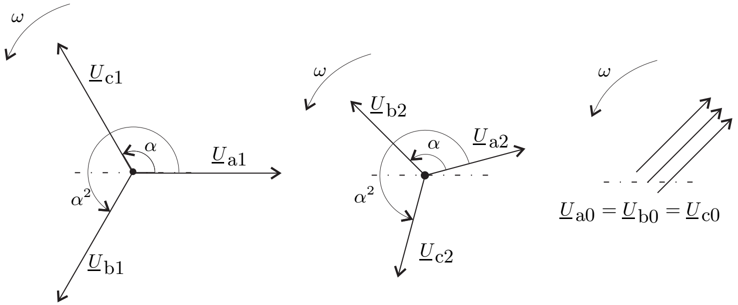
1 Main idea

A set of three unbalanced phasors can be decomposed into the sum of:

- three phasors making up a positive (or direct) sequence
- three phasors making up an negative sequence
- three phasors making up a zero sequence

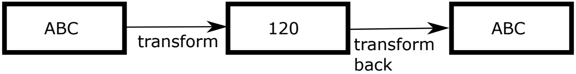
Thus, we end up with three 3-phase systems but each one of them is balanced (thus, easy to analyze). These are called the **symmetrical components**.

1 Symmetrical components



- **Positive sequence**: three rotating vectors, of same magnitude, shifted by 120° which an observer sees passing in the order a,b,c,a,b,c
- **Negative sequence**: three rotating vectors, of same magnitude, shifted by 120° which an observer sees passing in the order a,c,b,a,c,b
- **Zero sequence**: three rotating vectors, of same magnitude and in phase

1 Steps to perform analysis in symmetrical components



1. Transform ABC system to symmetrical components (this step is sometimes called “symmetrization”).
2. Carry out all computations in that framework
3. Transform back to ABC to get actual currents and voltages (this step is sometimes called de-symmetrization).

Simplifications can be attributed to the fact that the symmetrical components are the eigenvectors of the admittance matrix

1 120 $\rightarrow$ ABC

Projecting the ABC components to the 120 gives:

$$\underline{U}_a = \underline{U}_{a1} + \underline{U}_{a2} + \underline{U}_{a0} = \underline{U}_1 + \underline{U}_2 + \underline{U}_0$$
$$\underline{U}_b = \underline{U}_{b1} + \underline{U}_{b2} + \underline{U}_{b0} = \alpha^2\underline{U}_1 + \alpha\underline{U}_2 + \underline{U}_0$$
$$\underline{U}_c = \underline{U}_{c1} + \underline{U}_{c2} + \underline{U}_{c0} = \alpha\underline{U}_1 + \alpha^2\underline{U}_2 + \underline{U}_0$$

In matrix form:

$$\underbrace{\begin{pmatrix} \underline{U}_a \\ \underline{U}_b \\ \underline{U}_c \end{pmatrix}}_{\underline{U}_{abc}} = \underbrace{\begin{pmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{pmatrix}}_{\underline{\mathbf{T}}} \underbrace{\begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_0 \end{pmatrix}}_{\underline{U}_{120}}$$

1 Transformation matrix

To get back to the ABC framework, we use the transformation matrix $\underline{\mathbf{S}}$, which is calculated as the inverse of $\underline{\mathbf{T}}$. The definition of the eigenvectors, having elements of magnitude 1, results in the fact that all elements of the matrix $\underline{\mathbf{S}}$ have a magnitude of $1/3$.

$$\underline{\mathbf{T}} = \begin{pmatrix} 1 & 1 & 1 \\ \alpha^2 & \alpha & 1 \\ \alpha & \alpha^2 & 1 \end{pmatrix} \qquad \underline{\mathbf{S}} = \underline{\mathbf{T}}^{-1} = \frac{1}{3} \cdot \begin{pmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{pmatrix}$$

where $\alpha = e^{j \cdot 120^\circ} = \frac{-1+j\cdot\sqrt{3}}{2}$ and $\alpha^2 = e^{j \cdot 240^\circ} = \frac{-1-j\cdot\sqrt{3}}{2}$.

It can be easily shown that $|\alpha| = 1$ and $1 + \alpha + \alpha^2 = 0$

The inverse in matrix form is thus:

$$\underbrace{\begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_0 \end{pmatrix}}_{\underline{U}_{120}} = \underbrace{\frac{1}{3}\begin{pmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{pmatrix}}_{\underline{\mathbf{S}}} \underbrace{\begin{pmatrix} \underline{U}_a \\ \underline{U}_b \\ \underline{U}_c \end{pmatrix}}_{\underline{U}_{abc}}$$

Which gives:

$$\underline{U}_1 = \frac{1}{3}\left(\underline{U}_a + \alpha \underline{U}_b + \alpha^2 \underline{U}_c\right)$$

$$\underline{U}_2 = \frac{1}{3}\left(\underline{U}_a + \alpha^2 \underline{U}_b + \alpha \underline{U}_c\right)$$

$$\underline{U}_0 = \frac{1}{3}\left(\underline{U}_a + \underline{U}_b + \underline{U}_c\right)$$

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Note that in a balanced system, we have:

$$\begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} 1 & \alpha & \alpha^2 \\ 1 & \alpha^2 & \alpha \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \underline{U}_\mathrm{a} \\ \alpha^2 \underline{U}_\mathrm{a} \\ \alpha \underline{U}_\mathrm{a} \end{pmatrix} = \begin{pmatrix} \underline{U}_\mathrm{a} \\ 0 \\ 0 \end{pmatrix}$$

1 ABC $\rightarrow$ 120: Impedance

Assume a 3-phase coltage source feeding a 3-phase Y-connected load:

$$\begin{pmatrix} \underline{U}_a \\ \underline{U}_b \\ \underline{U}_c \end{pmatrix} = \begin{pmatrix} \underline{Z}_a & 0 & 0 \\ 0 & \underline{Z}_b & 0 \\ 0 & 0 & \underline{Z}_c \end{pmatrix} \begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix}$$

Converting to 120 sequence:

$$\underline{\mathbf{S}} \begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_0 \end{pmatrix} = \begin{pmatrix} \underline{Z}_a & 0 & 0 \\ 0 & \underline{Z}_b & 0 \\ 0 & 0 & \underline{Z}_c \end{pmatrix} \underline{\mathbf{S}} \begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_0 \end{pmatrix} \Leftrightarrow \begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_0 \end{pmatrix} = \underbrace{\underline{\mathbf{T}} \begin{pmatrix} \underline{Z}_a & 0 & 0 \\ 0 & \underline{Z}_b & 0 \\ 0 & 0 & \underline{Z}_c \end{pmatrix} \underline{\mathbf{S}}}_{\underline{\mathbf{Z}}_{120}} \begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_0 \end{pmatrix}$$

1 ABC $\rightarrow$ 120: Impedance

Assume a 3-phase coltage source feeding a 3-phase Y-connected load:

$$\begin{pmatrix} \underline{Z}_1 \\ \underline{Z}_2 \\ \underline{Z}_0 \end{pmatrix} = \frac{1}{3} \begin{pmatrix} \underline{Z}_a + \underline{Z}_b + \underline{Z}_c & \underline{Z}_a + \alpha^2 \underline{Z}_b + \alpha \underline{Z}_c & \underline{Z}_a + \alpha \underline{Z}_b + \alpha^2 \underline{Z}_c \\ \underline{Z}_a + \alpha \underline{Z}_b + \alpha^2 \underline{Z}_c & \underline{Z}_a + \underline{Z}_b + \underline{Z}_c & \underline{Z}_a + \alpha^2 \underline{Z}_b + \alpha \underline{Z}_c \\ \underline{Z}_a + \alpha^2 \underline{Z}_b + \alpha \underline{Z}_c & \underline{Z}_a + \alpha \underline{Z}_b + \alpha^2 \underline{Z}_c & \underline{Z}_a + \underline{Z}_b + \underline{Z}_c \end{pmatrix}$$

If the impedances are the same ($\underline{Z}_a = \underline{Z}_b = \underline{Z}_c$):

$$\begin{pmatrix} \underline{Z}_1 \\ \underline{Z}_2 \\ \underline{Z}_0 \end{pmatrix} = \begin{pmatrix} \underline{Z}_a \\ \underline{Z}_b \\ \underline{Z}_c \end{pmatrix}$$

1 Comments

- It should be noted that the admittance matrices in the ABC and 120 systems incorporate the same information, but are not equal.
- No zero-sequence components exist if the sum of the unbalanced phasors is zero.
- Zero sequence components are never present in the **line voltages** regardless of the degree of unbalance. Can you explain this?

ABC	120
$\underline{I}_{ABC} = \underline{T}\,\underline{I}_{120}$	$\underline{I}_{120} = \underline{S}\,\underline{I}_{ABC}$
$\underline{U}_{ABC} = \underline{T}\,\underline{U}_{120}$	$\underline{U}_{120} = \underline{S}\,\underline{U}_{ABC}$
$\underline{E}_{ABC} = \underline{T}\,\underline{E}_{120}$	$\underline{E}_{120} = \underline{S}\,\underline{E}_{ABC}$
$\underline{Z}_{ABC} = \underline{T}\,\underline{Z}_{120}\,\underline{S}$	$\underline{Z}_{120} = \underline{S}\,\underline{Z}_{ABC}\,\underline{T}$
$\underline{Y}_{ABC} = \underline{T}\,\underline{Y}_{120}\,\underline{S}$	$\underline{Y}_{120} = \underline{S}\,\underline{Y}_{ABC}\,\underline{T}$
$\underline{I}_{ABC} = \underline{Y}_{ABC}\,\underline{E}_{ABC}$	$\underline{I}_{120} = \underline{Y}_{120}\,\underline{E}_{120}$

2 Outline

2 Power equations

In ABC the three-phase apparent power is:

$$\underline{S}_{3\varphi} = \underline{U}_a \underline{I}_a^* + \underline{U}_b \underline{I}_b^* + \underline{U}_c \underline{I}_c^* = \underline{\mathbf{U}}\,\underline{\mathbf{I}}^*$$

Considering:

$$\begin{aligned} \underline{\mathbf{U}} &= \underline{\mathbf{T}}\,\underline{\mathbf{U}}_{120} \\ \underline{\mathbf{I}} &= \underline{\mathbf{T}}\,\underline{\mathbf{I}}_{120} \end{aligned}$$

We get:

$$\begin{aligned} \underline{S}_{3\varphi} &= \left(\underline{\mathbf{T}}\,\underline{\mathbf{U}}_{120}\right)^T \cdot \left(\underline{\mathbf{T}}\,\underline{\mathbf{I}}_{120}\right)^* = \\ &= \left(\underline{\mathbf{U}}_{120}\right)^T \cdot \underbrace{\underline{\mathbf{T}}^T \left(\underline{\mathbf{T}}\right)^*}_{3\,\mathbf{I_d}} \cdot \left(\underline{\mathbf{I}}_{120}\right)^* \end{aligned}$$

We can simplify this equation by calculating the middle part as:

$$\mathbf{T}^T \left(\underline{\mathbf{T}}\right)^* = \left(\left(\left(\underline{\mathbf{T}}\right)^*\right)^T \underline{\mathbf{T}}\right)^T = 3\left(\underline{\mathbf{T}}^{-1}\underline{\mathbf{T}}\right)^T = 3\,\mathbf{I_d}$$

where $\mathbf{I_d}$ represents the identity matrix.

2 Power equations

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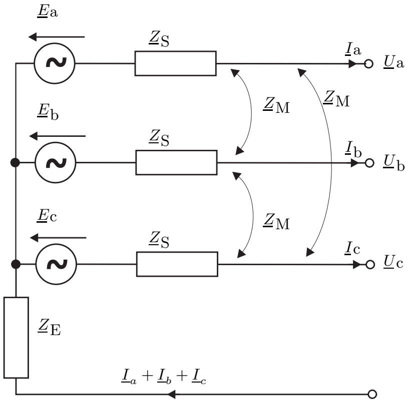
Leading to:

$$\underline{S}_{3\varphi} = 3 \cdot (\underline{\mathbf{U}}_{120})^T \cdot (\underline{\mathbf{I}}_{120})^* = \\ = 3 \cdot (\underline{U}_1 \underline{I}_1^* + \underline{U}_2 \underline{I}_2^* + \underline{U}_0 \underline{I}_0^*)$$

- A factor of 3 arises between the expressions of the powers in the ABC and symmetrical component systems.
- Each element of the ABC system carries triple the power of its equivalent in the symmetrical component system

3 Outline

Let's consider the following 3ϕ system:



where $\underline{Z}_S$ is the line impedance, $\underline{Z}_M$ is the mutual impedance between lines and $\underline{Z}_E$ in the neutral impedance.

Using KCL and KVL, we can write the equations of the system for phase *a*:

$$\underline{U}_a = -(\underline{I}_a + \underline{I}_b + \underline{I}_c)\underline{Z}_E + \underline{E}_a - \underline{Z}_S\underline{I}_a - \underline{Z}_M\underline{I}_b - \underline{Z}_M\underline{I}_c$$

Or, in matrix form for the entire system:

$$\begin{pmatrix} \underline{E}_a \\ \underline{E}_b \\ \underline{E}_c \end{pmatrix} = \begin{pmatrix} \underline{Z}_S + \underline{Z}_E & \underline{Z}_M + \underline{Z}_E & \underline{Z}_M + \underline{Z}_E \\ \underline{Z}_M + \underline{Z}_E & \underline{Z}_S + \underline{Z}_E & \underline{Z}_M + \underline{Z}_E \\ \underline{Z}_M + \underline{Z}_E & \underline{Z}_M + \underline{Z}_E & \underline{Z}_S + \underline{Z}_E \end{pmatrix} \cdot \begin{pmatrix} \underline{I}_a \\ \underline{I}_b \\ \underline{I}_c \end{pmatrix} + \begin{pmatrix} \underline{U}_a \\ \underline{U}_b \\ \underline{U}_c \end{pmatrix}$$

3 Balanced system

In a balanced, properly transposed system, we can simplify:

$$\begin{aligned}\underline{U}_a &= -(\underline{I}_a + \underline{I}_b + \underline{I}_c)\underline{Z}_E + \underline{E}_a - \underline{Z}_S\underline{I}_a - \underline{Z}_M(\underline{I}_b + \underline{I}_c) \\ &= -\underbrace{(\underline{I}_a + \underline{I}_b + \underline{I}_c)}_{0}\underline{Z}_E + \underline{E}_a - \underline{Z}_S\underline{I}_a - \underline{Z}_M\underbrace{(\underline{I}_b + \underline{I}_c)}_{-\underline{I}_a} \\ &= \underline{E}_a - \underline{I}_a(\underline{Z}_S - \underline{Z}_M) \\ &= \underline{E}_a - \underline{I}_a\underline{Z}_L\end{aligned}$$

Leading to:

$$\begin{pmatrix}\underline{E}_a \\ \underline{E}_b \\ \underline{E}_c\end{pmatrix} = \begin{pmatrix}\underline{Z}_L & 0 & 0 \\ 0 & \underline{Z}_L & 0 \\ 0 & 0 & \underline{Z}_L\end{pmatrix} \cdot \begin{pmatrix}\underline{I}_a \\ \underline{I}_b \\ \underline{I}_c\end{pmatrix} + \begin{pmatrix}\underline{U}_a \\ \underline{U}_b \\ \underline{U}_c\end{pmatrix}$$

where $\underline{Z}_L = \underline{Z}_S - \underline{Z}_M$.

Thus, we are able to analyze the system per-phase.

3 Unbalanced system

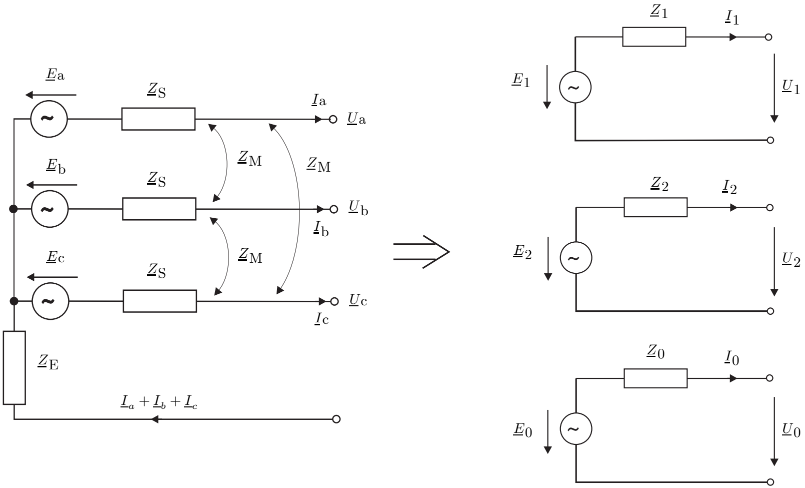
In an unbalanced system, the above approach does not work (why?). If we use the symmetrical components transformation (see slide 17) we can get:

$$\begin{pmatrix} \underline{E}_1 \\ \underline{E}_2 \\ \underline{E}_0 \end{pmatrix} = \begin{pmatrix} \underline{Z}_1 & 0 & 0 \\ 0 & \underline{Z}_2 & 0 \\ 0 & 0 & \underline{Z}_0 \end{pmatrix} \cdot \begin{pmatrix} \underline{I}_1 \\ \underline{I}_2 \\ \underline{I}_0 \end{pmatrix} + \begin{pmatrix} \underline{U}_1 \\ \underline{U}_2 \\ \underline{U}_0 \end{pmatrix} \tag{3.1}$$

where $\underline{Z}_1 = \underline{Z}_2 = \underline{Z}_S - \underline{Z}_M$ and $\underline{Z}_0 = \underline{Z}_S + 2\,\underline{Z}_M + 3\,\underline{Z}_E$.

✍ Even though the ABC system equations in an unbalanced system are not decoupled, the 120 equivalent equations are decoupled and we can analyze them per-phase.

Plotting the equivalent 120 circuits, gives:



3 Positive sequence system

- Corresponds to the single-phase equivalent circuit of the balanced three-phase system. The generator voltage feeds the circuit in the positive sequence system. This voltage is equal to the voltage $\underline{E}_{\mathrm{a}}$ of a symmetrically-operated generator.
- The impedances of the passive elements of the positive sequence system are included in the impedance $\underline{Z}_1$. It should be noted that $\underline{Z}_1$ is independent of the neutral-to-ground impedance $\underline{Z}_{\mathrm{E}}$.
- The grounding of the neutrals is irrelevant since the sum of the currents is zero. Delta-connected elements must be transformed into wye connections. In the symmetric, three-phase circuit, all neutral points have the same potential; it does not matter whether or not they are connected. Thus, all neutral points of the equivalent, positive sequence circuit can be thought of as connected.

3 Negative sequence system

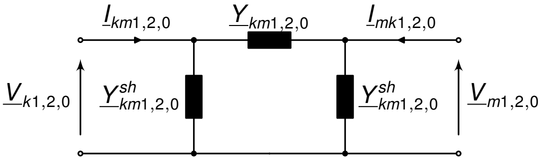
- It is derived in a manner analogous to that of the positive sequence system. However, the voltage source component of the generator voltage is zero so that no supply voltage normally exists in the circuit.
- In the passive part of the network, the impedance $\underline{Z}_2$ is equal to $\underline{Z}_1$. This is due to the fact that the neutral point grounding has no effect on the negative sequence system (therefore, we can set $\underline{Z}_2 = \underline{Z}_1$).
- In the equivalent circuit of the negative sequence system, delta-wye transformations must be performed and all neutral points must be connected to one another.

3 Zero sequence system

- It is fed by the zero sequence component of the generator voltage, which is zero for symmetrical generators.
- The zero sequence impedance $\underline{Z}_0$ of the passive elements must be included. This impedance generally differs from the positive and negative sequence system impedances.
- The treatment of the neutral points is very important in the zero sequence system. As mentioned previously, zero sequence currents can **only** flow through neutral point connections. The connections in the zero sequence diagram correspond to the grounding conditions in the real physical system.
- Impedances at the neutral point connections must be included with **triple the value of the physical impedance**. The threefold value is necessary because triple the real zero sequence current actually flows through the neutral ground connection.

4 Outline

4 Line model



Positive, Negative, and Zero sequence model

$$\begin{bmatrix} \underline{I}_{km1,2,0} \\ \underline{I}_{mk1,2,0} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{km1,2,0} + \underline{Y}^{sh}_{km1,2,0} & -\underline{Y}_{km1,2,0} \\ -\underline{Y}_{km1,2,0} & \underline{Y}_{km1,2,0} + \underline{Y}^{sh}_{km1,2,0} \end{bmatrix} \begin{bmatrix} \underline{V}_{k1,2,0} \\ \underline{V}_{m1,2,0} \end{bmatrix}$$

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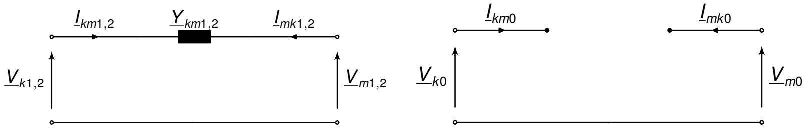
If the model is of type Yy, YNy, Yd, Dd:



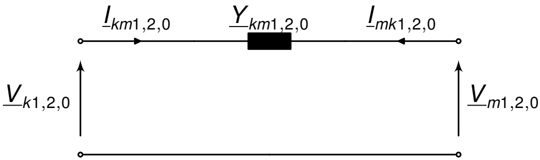
Positive & Negative sequence

Zero sequence model

$$\begin{bmatrix} \underline{I}_{km1,2} \\ \underline{I}_{mk1,2} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{km1,2} & -\underline{Y}_{km1,2} \\ -\underline{Y}_{km1,2} & \underline{Y}_{km1,2} \end{bmatrix} \begin{bmatrix} \underline{V}_{k1,2} \\ \underline{V}_{m1,2} \end{bmatrix}$$

$$\begin{bmatrix} \underline{I}_{km0} \\ \underline{I}_{mk0} \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{V}_{k0} \\ \underline{V}_{m0} \end{bmatrix}$$

If the model is of type YNyn:

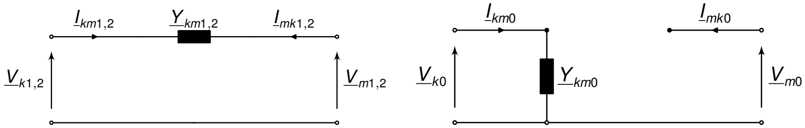


Positive & Negative & Zero sequence

$$\begin{bmatrix} \underline{I}_{km1,2,0} \\ \underline{I}_{mk1,2,0} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{km1,2,0} & -\underline{Y}_{km1,2,0} \\ -\underline{Y}_{km1,2,0} & \underline{Y}_{km1,2,0} \end{bmatrix} \begin{bmatrix} \underline{V}_{k1,2,0} \\ \underline{V}_{m1,2,0} \end{bmatrix}$$

4 Transformer model

If the model is of type YNd:



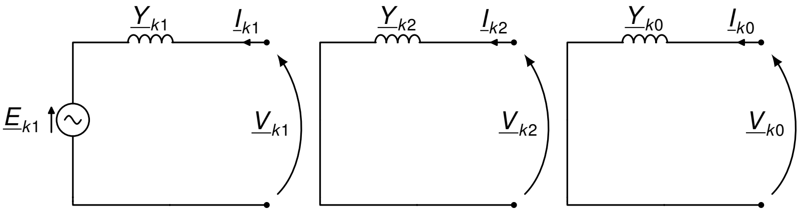
Positive & Negative sequence

Zero sequence model

$$\begin{bmatrix} \underline{I}_{km1,2} \\ \underline{I}_{mk1,2} \end{bmatrix} = \begin{bmatrix} \underline{Y}_{km1,2} & -\underline{Y}_{km1,2} \\ -\underline{Y}_{km1,2} & \underline{Y}_{km1,2} \end{bmatrix} \begin{bmatrix} \underline{V}_{k1,2} \\ \underline{V}_{m1,2} \end{bmatrix}$$

$$\begin{bmatrix} \underline{I}_{km0} \\ \underline{I}_{mk0} \end{bmatrix} = \begin{bmatrix} Y_{km0} & 0 \\ 0 & 0 \end{bmatrix} \begin{bmatrix} \underline{V}_{k0} \\ \underline{V}_{m0} \end{bmatrix}$$

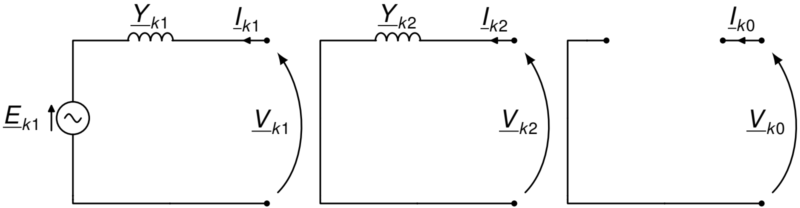
If the model is of type YN:



We use the load assumption for uniformity and the Norton equivalent:

$$
\begin{aligned}
\underline{I}_{k1} &= \underline{Y}_{k1}\underline{V}_{k1} - \underline{Y}_{k1}\underline{E}_{k1} \\
\underline{I}_{k2,0} &= \underline{Y}_{k2,0}\underline{V}_{k2,0}
\end{aligned}
$$

If the model is of type Y or D:



We use the load assumption for uniformity and the Norton equivalent:

$$\underline{I}_{k1} = \underline{Y}_{k1}\underline{V}_{k1} - \underline{Y}_{k1}\underline{E}_{k1}$$
$$\underline{I}_{k2} = \underline{Y}_{k2}\underline{V}_{k2}$$
$$\underline{I}_{k0} = 0$$