EEN452 - Control and Operation of Electric Power Systems

Part 2A: Synchronous machine model (simplified)

https://sps.cut.ac.cy/courses/een452/

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Today's learning objectives

After this part of the lecture and additional reading, you should be able to ...

1. ...define the simplified synchronous machine model;
2. ...understand the electromechanical interactions of a synchronous machine in steady-state.

Much of the material was adapted from the courses delivered by Prof. Thierry Van Cutsem at the University of Liege.

Why synchronous machines?

- produce the major part of the electric energy
  - range from a few kVA to a few hundred MVA
  - the biggest are rated 1500 MVA
- play an important role:
  - they impose the frequency of sinusoidal voltages and currents
  - they provide an ”energy buffer” (through the kinetic energy stored in their rotating masses)
  - they can produce or consume reactive power (needed to regulate voltage).

1 Outline

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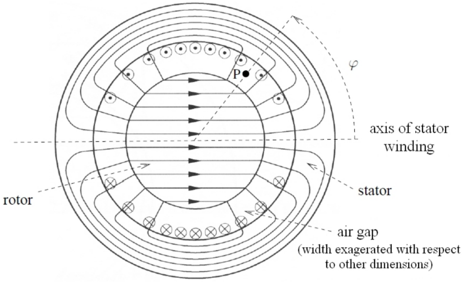
1 Magnetic field created by the stator

- stator (or armature) = motionless, separated from the rotor by a small air gap
- subjected to varying magnetic flux → built up of thin laminations to decrease eddy (or Foucault) currents
- equipped with three windings, distributed 120 degrees apart in space.

Magnetic field created by a direct current flowing in one of the stator windings:

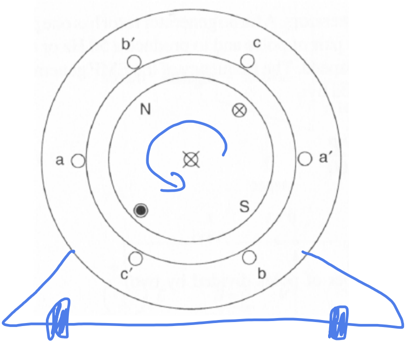


1 Magnetic field created by the stator

The magnetic field lines cross the air gap radially. The amplitude $B(\varphi)$ of the magnetic flux density at point P:

- is a periodic function of φ with period 2π
- this function has a ”staircase” shape
- is made as close as possible to a sinusoid, by properly distributing the conductors along the air gap.

Layout of the three phases (each winding is represented by a single turn for clarity):



1 Magnetic field created by the stator

Total flux density created by the three phases at point P corresponding to angle φ:

$$B_{3\varphi}(\varphi) = ki_a\cos(\varphi) + ki_b\cos(\varphi - \frac{2\pi}{3}) + ki_c\cos(\varphi - \frac{4\pi}{3})$$

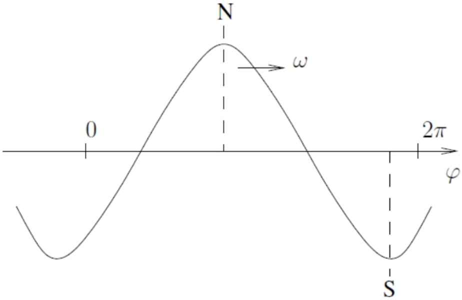
If three-phase alternating currents are flowing in the windings:

$$\begin{aligned}
B_{3\varphi}(\varphi) =& \sqrt{2}kI\Big[\cos(\omega t + \psi)\cos(\varphi) + \cos(\omega t + \psi - \frac{2\pi}{3})\cos(\varphi - \frac{2\pi}{3}) \\
&+ \cos(\omega t + \psi - \frac{4\pi}{3})\cos(\varphi - \frac{4\pi}{3})\Big] \\
=& \frac{\sqrt{2}kI}{2}\Big[\cos(\omega t + \psi + \varphi) + \cos(\omega t + \psi - \varphi) + \cos(\omega t + \psi + \varphi - \frac{4\pi}{3}) \\
&+ \cos(\omega t + \psi - \varphi) + \cos(\omega t + \psi + \varphi - \frac{2\pi}{3}) + \cos(\omega t + \psi - \varphi)\Big] \\
=& \frac{3\sqrt{2}kI}{2}\cos(\omega t + \psi - \varphi)
\end{aligned}$$

This is the equation of a wave rotating in the air gap at the angular speed ω

1 Magnetic field created by the stator

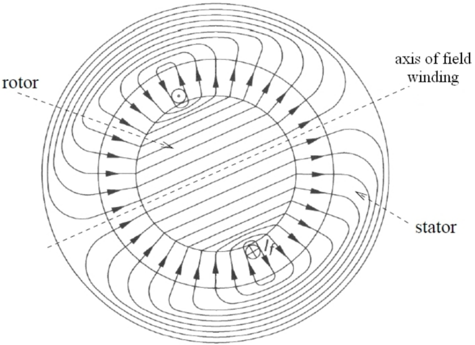
If we ”unroll” the air-gap:



- The three-phase alternating currents all together produce the same magnetic field as a magnet (or a coil carrying a direct current) rotating at the angular speed ω
- North pole of magnet $\rightarrow$ maximum of $B(\varphi)$
- South pole of magnet $\rightarrow$ minimum of $B(\varphi)$

1 Magnetic field created by the rotor

Magnetic field created by this direct current (field winding represented by a single turn for clarity):



- rotor = rotating part, separated from the rotor by the air gap
- carries a winding in which a direct current flows, in steady-state operation
- referred to as field winding

1 Machines with multiple pairs of poles

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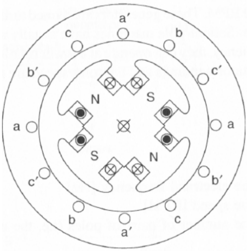
Some turbines operate at a lower speed but AC voltages and currents at the stator must keep the same period $T = \frac{1}{f}$

- the rotor carries p **pairs of poles**
- during a period T, the rotor makes only $\frac{1}{p}$ of a whole revolution
- the stator carries p sets of (a, b, c) windings
- one winding spans an angle of π/p radians
- during a period T, each stator winding is still swept by one North and one South pole of rotor



example for $p = 2$

- speed: $\frac{60 \cdot f}{p} rpm$
- The various windings relative to a given phase are connected (in series or parallel) to end up with a three-phase machine.

2 Outline

2 Round-rotor generators (or turbo-alternators)

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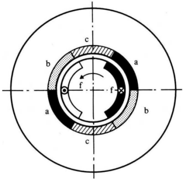
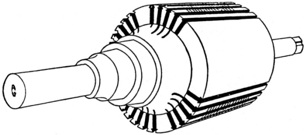
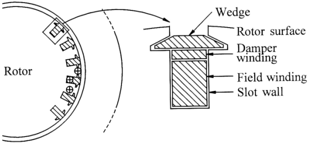
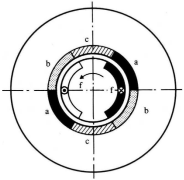


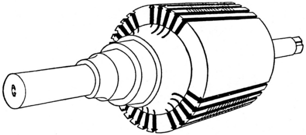


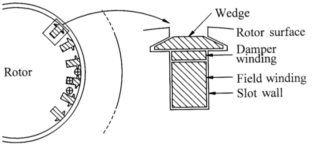


- Driven by steam or gas turbines, which rotate at high speed
- $p = 1$ (conventional thermal units) or $p = 2$ (nuclear units)
- cylindrical rotor made up of solid steel forging
- diameter $<<$ length (centrifugal force)

2 Round-rotor generators (or turbo-alternators)







- field winding made up of conductors distributed on the rotor, in milled slots
- even if the generator efficiency is around 99%, the heat produced by Joule losses has to be evacuated.
- Large generators are cooled by hydrogen (heat evacuation 7 times better than air) or water (12 times better) flowing in the hollow stator conductors.

2 Round-rotor generators (or turbo-alternators)

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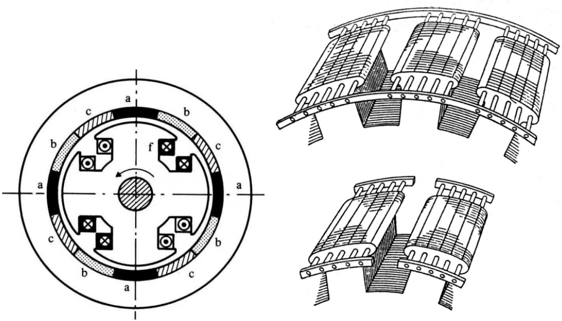
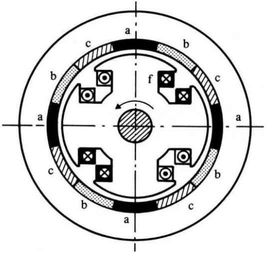
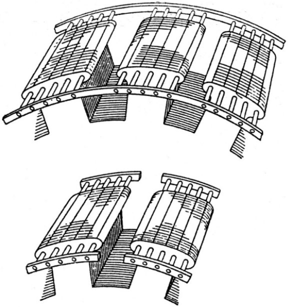


- Driven by hydraulic turbines (or diesel engines), which rotate at low speed
- p is much higher (at least 4) $\rightarrow$ it is more convenient to have field windings concentrated and placed on the poles
- air gap is not constant: min. in front of a pole, max. in between two poles

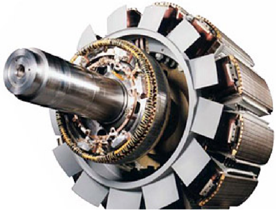
2 Salient-pole generators

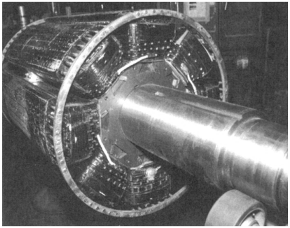


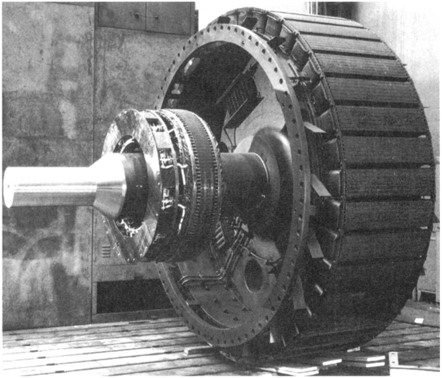


- poles are shaped to also minimize space harmonics (see slide 6)
- diameter $>>$ length (to have space for the many poles)
- rotor is laminated (poles easier to construct)
- generators usually cooled by the flow of air around the rotor

2 Salient-pole generators





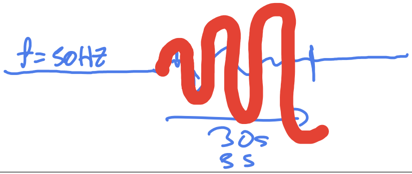


3 Outline

3 Number of windings

There are typically 5 physical windings on a synchronous machine:

- 3 stator windings (a-phase, b-phase, and c-phase)
- 1 main field winding
- Damper or Amortisseur windings on the pole-faces
  - **round-rotor machines**: copper/brass bars placed in the same slots at the field winding, and interconnected to form a damper cage (similar to the squirrel cage of an induction motor)
  - **salient-pole machines**: copper/brass rods embedded in the poles and connected at their ends to rings or segments
  - They can be continuous or noncontinuous (see fig. in slide 15)



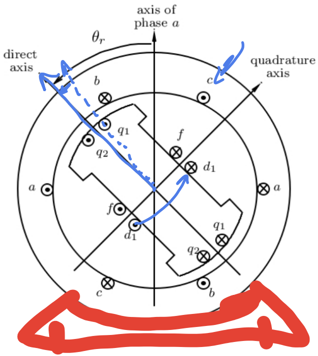
3 Damper windings

- **in perfect steady state**: the magnetic fields produced by both the stator and the rotor are fixed relative to the rotor $\rightarrow$ no current induced in dampers[1]
- **after a disturbance**: the rotor moves with respect to stator magnetic field $\rightarrow$ currents are induced in the dampers...
  ... which, according to Lenz's law, create a damping torque helping the rotor to align on the stator magnetic field
- **Round-rotor generators**: the solid rotor offers a path for eddy currents, which produce an effect similar to those of amortisseurs.

[1] Amortisseur means "dead"

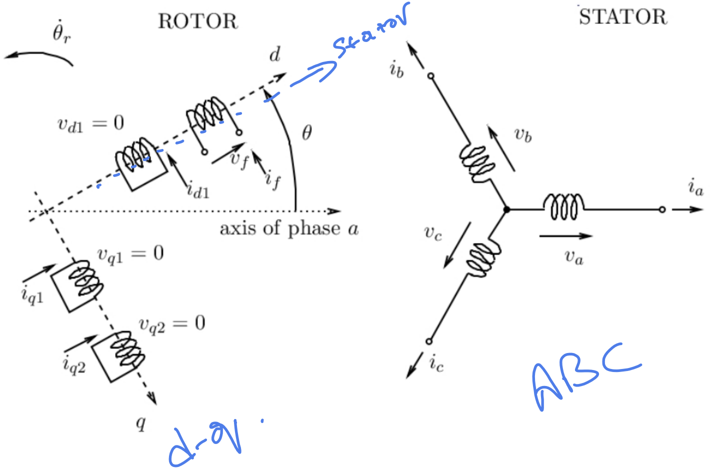
3 Modeled windings

Number of rotor windings = degree of sophistication of model. But more detailed model → more data are needed while measurement devices can be connected only to the field winding.



- 3 stator windings
- Most widely used model: 3 or 4 rotor windings:
  - f: field winding, d_1, q_1: damper windings
  - q_2: accounts for eddy currents in rotor – not used in (laminated) salient-pole generators

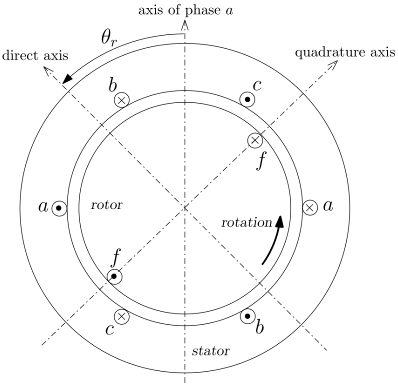
Detailed model (next lesson):



4 Outline

4 Simplifying assumptions

- round rotor
- saturation of magnetic material neglected
- on the rotor: field winding only (acceptable since focus is on steady-state operation)
- single pair of poles (does not affect the electrical behaviour)



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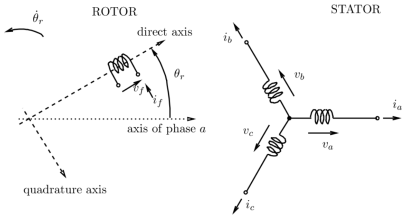

Stator:

$$v_a = -R_a i_a - \frac{d\psi_a}{dt} \quad v_b = -R_b i_b - \frac{d\psi_b}{dt} \quad v_c = -R_c i_c - \frac{d\psi_c}{dt}$$

R_a : Resistance of each phase ψ_a, ψ_b and ψ_c : flux linkages in phases

Field winding:

$$v_f = -R_f i_f - \frac{d\psi_f}{dt}$$

R_f : Resistance of winding ψ_f : flux linkages in winding

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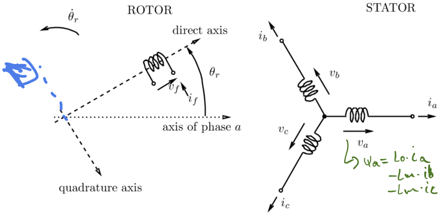

- The magnetic circuit and all rotor winding circuits are symmetrical with respect to the polar and inter-polar (between-poles) axes
- We give these axes special names:
  - Polar axis: Direct, or d-axis
  - Interpolar axis: Quadrature, or q-axis
- q-axis is 90^o from the d-axis but can be modeled both as *leading* or *lagging*. Both assumptions are correct and used by textbooks. $\rightarrow$ in this course, we assume **lagging**.

4 Inductance matrix

$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \end{bmatrix} = \underbrace{\begin{bmatrix} L_o & -L_m & -L_m & L_{af}\cos\theta_r \\ -L_m & L_o & -L_m & L_{af}\cos(\theta_r - \frac{2\pi}{3}) \\ -L_m & -L_m & L_o & L_{af}\cos(\theta_r - \frac{4\pi}{3}) \\ L_{af}\cos\theta_r & L_{af}\cos(\theta_r - \frac{2\pi}{3}) & L_{af}\cos(\theta_r - \frac{4\pi}{3}) & L_{ff} \end{bmatrix}}_{\mathbf{L}(\theta_r)} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix}$$

where $L_o, L_m > 0$.

- Self-inductance of any stator winding is constant (due to round rotor)
- mutual inductance between any two phases is constant (due to round rotor)
- . . . and negative since a positive current i_x in phase x creates a negative flux ψ_y in phase y ($x \neq y$)

4 Inductance matrix

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$$\begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \end{bmatrix} = \underbrace{\begin{bmatrix} L_o & -L_m & -L_m & L_{af}\cos\theta_r \\ -L_m & L_o & -L_m & L_{af}\cos(\theta_r - \frac{2\pi}{3}) \\ -L_m & -L_m & L_o & L_{af}\cos(\theta_r - \frac{4\pi}{3}) \\ L_{af}\cos\theta_r & L_{af}\cos(\theta_r - \frac{2\pi}{3}) & L_{af}\cos(\theta_r - \frac{4\pi}{3}) & L_{ff} \end{bmatrix}}_{\mathbf{L}(\theta_r)} \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix}$$

where $L_o, L_m > 0$.

- self-inductance of field winding is constant (path of magnetic field identical whatever the position of the rotor)
- mutual inductance between one phase and the field winding is maximum and positive when $\theta_r = 0$, zero when $\theta_r = \frac{\pi}{2}$, minimum and negative when $\theta_r = \pi$

5 Fundamental equations

- rotation speed equal to nominal angular frequency:

$$\dot{\theta}_r = \omega_N \quad \theta_r = \theta_r^o + \omega_N t$$

θ_r^0: rotor position at $t = 0$

- constant direct current in field winding: $i_f = I_f$
- balanced three-phase voltages and currents in stator:

$$\begin{aligned}
v_a(t) &= \sqrt{2} V \cos(\omega_N t + \theta) & i_a(t) &= \sqrt{2} I \cos(\omega_N t + \psi) \\
v_b(t) &= \sqrt{2} V \cos(\omega_N t + \theta - \frac{2\pi}{3}) & i_b(t) &= \sqrt{2} I \cos(\omega_N t + \psi - \frac{2\pi}{3}) \\
v_c(t) &= \sqrt{2} V \cos(\omega_N t + \theta - \frac{4\pi}{3}) & i_c(t) &= \sqrt{2} I \cos(\omega_N t + \psi - \frac{4\pi}{3})
\end{aligned}$$

with the corresponding phasors:

$$\underline{V} = V e^{j\theta} \qquad \underline{I} = I e^{j\psi}$$

5 Flux linkage in one stator winding (phase a)

$$\psi_a = L_o\sqrt{2}I\cos(\omega_N t+\psi) - L_m\sqrt{2}I\cos(\omega_N t+\psi-\frac{2\pi}{3}) - L_m\sqrt{2}I\cos(\omega_N t+\psi-\frac{4\pi}{3}) + L_{af}\cos(\omega_N t+\theta_r^o)I_f$$

Adding and subtracting $L_m\sqrt{2}I\cos(\omega_N t+\psi)$ yields:

$$\begin{aligned}\psi_a =& L_o\sqrt{2}I\cos(\omega_N t+\psi) + L_m\sqrt{2}I\cos(\omega_N t+\psi)\\ &- L_m\sqrt{2}I\underbrace{\left(\cos(\omega_N t+\psi)+\cos(\omega_N t+\psi-\frac{2\pi}{3})+\cos(\omega_N t+\psi-\frac{4\pi}{3})\right)}_{0}\\ &+ L_{af}I_f\cos(\omega_N t+\theta_r^o)\\ =& \underbrace{\sqrt{2}(L_o+L_m)I\cos(\omega_N t+\psi)}_{\psi_a^s} + \underbrace{L_{af}I_f\cos(\omega_N t+\theta_r^o)}_{\psi_a^r}\end{aligned}$$

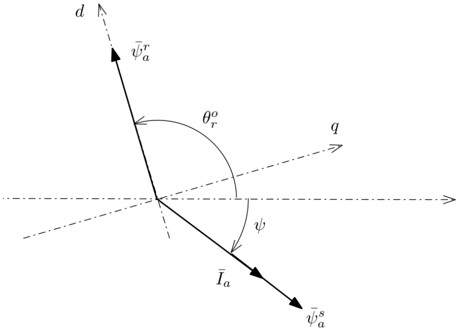
ψ_a^s: flux of the rotating field produced by the three stator currents
ψ_a^r: flux of the field created by the current i_f

5 Flux linkage in one stator winding (phase a)

Both flux components being sinusoidal functions of time (with angular frequency ω_N), they can be characterized by phasors:

$$\underline{\psi}_a^s = (L_o + L_m) I e^{j\psi} \quad \underline{\psi}_a^r = \frac{L_{af}}{\sqrt{2}} I_f e^{j\theta_r^o}$$

Phasor diagram:



Horizontal axis
= axis on which rotating vectors are projected
= axis to which the rotor position is referred, i.e. axis of phase a

5 Flux linkage in field winding

$$\psi_f = L_{ff} I_f + L_{af}\cos(\omega_N t + \theta_r^o)\sqrt{2} I \cos(\omega_N t + \psi)$$
$$+ L_{af}\cos(\omega_N t + \theta_r^o - \frac{2\pi}{3})\sqrt{2} I \cos(\omega_N t + \psi - \frac{2\pi}{3})$$
$$+ L_{af}\cos(\omega_N t + \theta_r^o - \frac{4\pi}{3})\sqrt{2} I \cos(\omega_N t + \psi - \frac{4\pi}{3})$$
$$= L_{ff} I_f + \frac{\sqrt{2}L_{af}}{2} I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi)\right]$$
$$+ \frac{\sqrt{2}L_{af}}{2} I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi - \frac{4\pi}{3})\right]$$
$$+ \frac{\sqrt{2}L_{af}}{2} I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi + \frac{4\pi}{3})\right]$$
$$= \underbrace{L_{ff} I_f}_{\psi_f^r} + \underbrace{\frac{3\sqrt{2}L_{af}}{2} I \cos(\theta_r^o - \psi)}_{\psi_f^s}$$

ψ_f^s: flux of the rotating field produced by the three stator currents; constant magnitude; at an angle $\theta_r^o - \psi$ wrt to field winding
ψ_f^r: flux created by field current

https://en.wikipedia.org/wiki/List_of_trigonometric_identities

5 Voltage-current relation at stator

Replacing v_a, i_a, and ψ_a by their expressions:

$$\sqrt{2}V\cos(\omega_N t+\theta) = -R_a\sqrt{2}I\cos(\omega_N t+\psi)+\sqrt{2}\omega_N(L_o+L_m)I\sin(\omega_N t+\psi)$$
$$+\sqrt{2}\frac{\omega_N L_{af}}{\sqrt{2}}I_f\sin(\omega_N t+\theta_r^o)$$

Let's define:

- $X=\omega_N(L_o+L_m)$: the synchronous reactance of the machine
- $E_q=\frac{\omega_N L_{af}}{\sqrt{2}}I_f$: RMS value of an e.m.f. proportional to field current I_f

The above equation becomes:

$$\sqrt{2}V\cos(\omega_N t+\theta) = -R_a\sqrt{2}I\cos(\omega_N t+\psi)+\sqrt{2}XI\cos(\omega_N t+\psi-\frac{\pi}{2})$$
$$+\sqrt{2}E_q\cos(\omega_N t+\theta_r^o-\frac{\pi}{2})$$

5 Voltage-current relation at stator

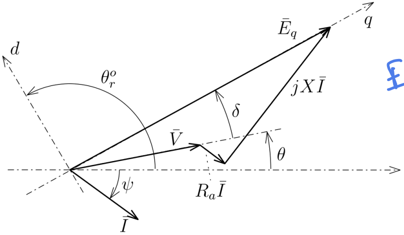
The corresponding phasor equation is:

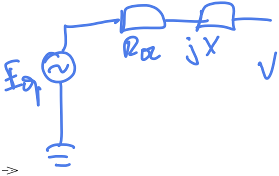
$$Ve^{j\theta} = -R_a I e^{j\psi} + XIe^{j\psi} e^{-j\frac{\pi}{2}} + E_q e^{j(\theta_r^o - \frac{\pi}{2})}$$

or, simply:

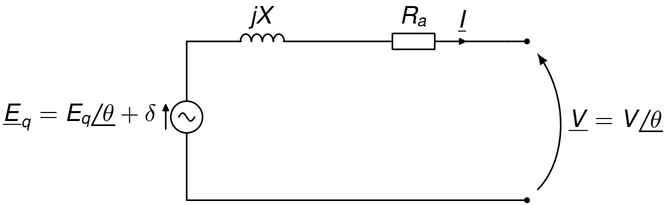
$$\underline{V} = -R_a \underline{I} - jX\underline{I} + \underline{E}_q$$

where $\underline{E}_q = E_q e^{j(\theta_r^o - \frac{\pi}{2})}$ is the phasor of the e.m.f. E_q, lying on the q axis





5 Per-phase equivalent circuit



- The synchronous reactance X characterizes the steady-state operation of the machine
- δ is the phase shift between the internal e.m.f. $\underline{E}_q$ and the terminal voltage $\underline{V}$
- δ is called the internal angle, load angle, or power angle of the machine

5 Nominal values and orders of magnitude

- Nominal voltage V_N: voltage for which the machine has been designed (in particular its insulation).
  The real voltage may deviate from this value by a few %
- nominal current I_N: current for which machine has been designed (in particular the cross-section of its conductors).
  Maximum current that can be accepted without limit in time
- nominal apparent power $S_N = \sqrt{3} V_N I_N$

The machine parameters in per-unit on the base ($S_B = S_N$, $V_B = V_N/\sqrt{3}$):

- $R_a \approxeq 0.005$ pu
- $X \approxeq 1.5 - 2.5$ pu (for a round-rotor machine)

6 Outline

6 Power balance of the stator

$$p_{r\rightarrow s} = p_T + p_{Js} + \frac{dW_{ms}}{dt}$$

where

- $p_{r\rightarrow s}$: power transfer from rotor to stator
- p_T: three-phase instantaneous power leaving the stator
- p_{Js}: Joule losses in stator windings
- W_{ms}: magnetic energy stored in the stator windings

The nature of $p_{r\rightarrow s}$

- mechanical power for sure (torque applied to rotating masses)
- is there some electromagnetic transfer of power (like in a transformer)?

6 Power balance of the rotor

$$p_f + P_m = p_{Jf} + \frac{dW_{mf}}{dt} + \frac{dW_c}{dt} + p_{r \to s}$$

where

- P_m: mechanical power provided by the turbine
- p_f: electrical power provided to the field winding by the excitation system
- p_{Jf}: Joule losses in the field winding
- W_{mf}: magnetic energy stored in the field winding
- W_c: kinetic energy of all rotating masses (generator and turbine)

Total electromagnetic energy stored in the machine:

$$W_{m,tot} = \frac{1}{2} \begin{bmatrix} i_a & i_b & i_c & i_f \end{bmatrix} \boldsymbol{L}(\theta_r) \begin{bmatrix} i_a \\ i_b \\ i_c \\ i_f \end{bmatrix} = \frac{1}{2} \begin{bmatrix} i_a & i_b & i_c & i_f \end{bmatrix} \begin{bmatrix} \psi_a \\ \psi_b \\ \psi_c \\ \psi_f \end{bmatrix}$$

$$= \underbrace{\frac{1}{2}(i_a\psi_a + i_b\psi_b + i_c\psi_c)}_{W_{ms}} + \underbrace{\frac{1}{2} i_f \psi_f}_{W_{mf}}$$

6 Motion equation

$$J\frac{d^2\theta_r}{dt^2} = T_m - T_e$$

where
J: moment of inertia of all rotating masses
T_m: mechanical torque applied to the rotor by the turbine
T_e: electromagnetic torque applied to the rotor by the generator
Multiplying by the rotor speed $d\theta_r/dt$:

$$J\frac{d\theta_r}{dt}\frac{d^2\theta_r}{dt^2} = \frac{d\theta_r}{dt}T_m - \frac{d\theta_r}{dt}T_e$$

$$\Leftrightarrow \frac{dW_c}{dt} = P_m - \frac{d\theta_r}{dt}T_e$$

and the power balance of the rotor becomes:

$$p_f + \frac{d\theta_r}{dt}T_e = p_{Jf} + \frac{dW_{mf}}{dt} + p_{r\rightarrow s}$$

6 Power balance of stator

$$\frac{1}{2} i_a \psi_a = (L_o + L_m) I^2 \cos^2(\omega_N t + \psi) + \frac{\sqrt{2}}{2} L_{af} I_f I \cos(\omega_N t + \theta_r^o) \cos(\omega_N t + \psi)$$
$$= \frac{1}{2}(L_o + L_m) I^2 + \frac{1}{2}(L_o + L_m) I^2 \cos(2\omega_N t + 2\psi) +$$
$$\frac{\sqrt{2}}{4} L_{af} I_f I \left[\cos(\theta_r^o - \psi) + \cos(2\omega_N t + \theta_r^o + \psi)\right]$$

By doing the same derivation for phases b and c, and adding all three results:

$$W_{ms} = \frac{1}{2}(i_a\psi_a + i_b\psi_b + i_c\psi_c) = \frac{3}{2}(L_o + L_m) I^2 + \frac{3\sqrt{2}}{4} L_{af} I_f I \cos(\theta_r^o - \psi)$$

$$W_{ms} \text{ is constant, i.e. } \frac{dW_{ms}}{dt} = 0$$

https://en.wikipedia.org/wiki/List_of_trigonometric_identities

6 Power balance of stator

In three-phase balanced operation:

$$p_T = 3P$$

where P is the active power produced by one phase.

Hence, the power balance of the stator simply becomes :

$$p_{r\rightarrow s} = 3P + p_{Js}$$

6 Power balance of rotor

$$W_{mf} = \frac{1}{2} i_f \psi_f = \frac{1}{2} L_{ff} I_f^2 + \frac{3\sqrt{2}}{4} L_{af} I\, I_f \cos(\theta_r^o - \psi)$$

$$W_{mf} \text{ is constant, i.e. } \frac{dW_{mf}}{dt} = 0$$

$$\frac{d\psi_f}{dt} = 0 \quad \Rightarrow \quad V_f = R_f I_f \quad \Rightarrow \quad p_f = R_f I_f^2 = p_{Jf}$$

In steady state, the power entering the field winding is dissipated in Joule losses!

The field current aims at “magnetizing” the rotor, allowing the torque T_e to be created, but the field winding does not exchange power with the other windings.

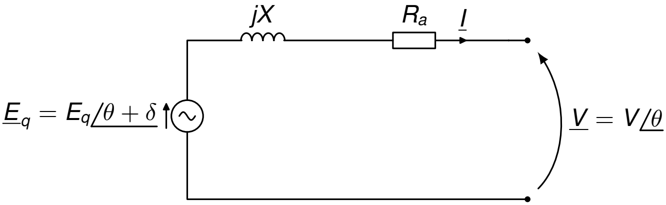
$$\frac{d\theta_r}{dt} = \omega_N \quad \frac{dW_c}{dt} = 0 \quad T_m = T_e \quad P_m = \omega_N T_e = \omega_N T_m$$

Hence, the power balance of the rotor simply becomes:

$$p_{r \to s} = \omega_N T_e = \omega_N T_m = P_m$$

where power $p_{r \to s}$ transfered from rotor to stator is purely mechanical!

6 Expression of active and reactive powers



Assuming $R_a \approx 0$, active and reactive power in per-unit can be given as:

$$P = -\frac{VE_q}{X}\sin(\theta - (\theta + \delta)) = \frac{VE_q}{X}\sin(\delta)$$

$$Q = -\frac{V^2}{X} + \frac{VE_q}{X}\cos(\theta - (\theta + \delta)) = -\frac{V^2}{X} + \frac{VE_q}{X}\cos(\delta)$$

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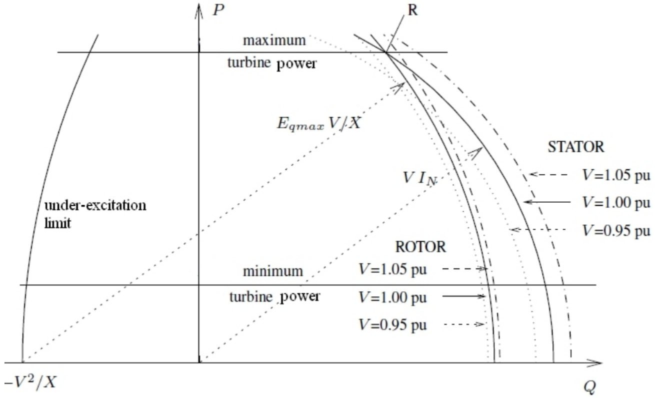
7 Capability curves

Seen from the network, a generator is characterized by three variables: V, P and Q

The capability curves define the set of admissible operating points in the (P, Q) space, ***under constant voltage*** V (justified by automatic voltage regulator)



Stator (heating) limit

$$\text{stator current } I = I_N \quad \text{in per-unit: } S^2 = P^2 + Q^2 = V^2 I_N^2$$

Rotor (heating) limit

$$\text{field current } I_f = I_{fmax} \quad \Rightarrow E_q = E_{qmax} = \frac{\omega_N L_{af}}{\sqrt{2}} I_{fmax}$$

With the same simplifying assumptions as before, and with $R_a = 0$:

$$P = \frac{E_{qmax} V}{X} \sin(\delta) \quad Q = \frac{E_{qmax} V}{X} \cos(\delta) - \frac{V^2}{X}$$

after eliminating δ:

$$\left(\frac{V E_{qmax}}{X}\right)^2 = \left(Q + \frac{V^2}{X}\right)^2 + P^2$$

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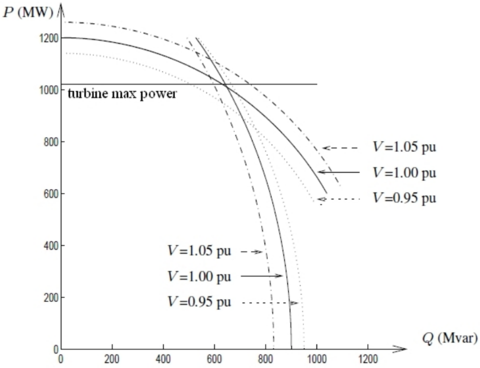
- Lower limit on active power caused by stability of combustion in thermal power plants
- maximum reactive power *in*creases when the active power *de*creases
  - to relieve an overloaded machine, P can be decreased but this power has to be produced by some other generators
- for a given value of P, the maximum reactive power increases with V
  - this holds true under the simplifying assumption of a non saturated machine; see next slide for a case with saturation
- in practice, under $V = 1$ pu, the two-by-two intersection points of respectively the turbine, the rotor and the stator limits are close to each other ("coherent" design of stator and rotor)
- the stator limits can be increased by a stronger cooling (e.g., higher hydrogen pressure in stator windings)

Under-excitation limit

Corresponds to a stability, not a thermal limit: absorbing more $Q \Rightarrow$ decreasing $E_q \Rightarrow$ decreasing $i_f \rightarrow$ maximum torque T_e decreases $\Rightarrow$ risk of losing synchronism.

7 Capability curves

Capability curves ($Q > 0$ part only) of a real machine with saturation taken into account



- the overall shape of the curves is the same
- but the rotor limit becomes more constraining when V increases