EEN452 - Control and Operation of Electric Power Systems

Part 5: Economics of electricity generation

https://sps.cut.ac.cy/courses/een452/

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Today's learning objectives

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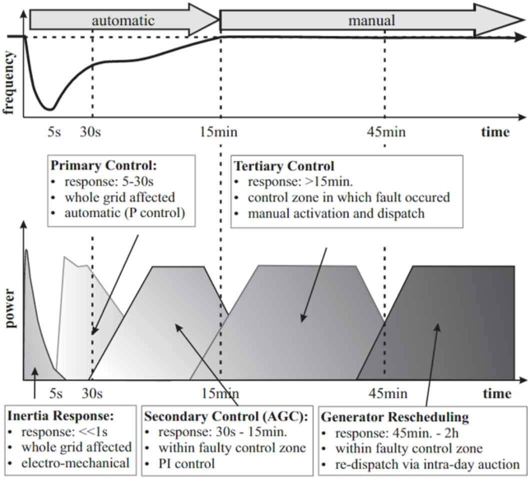
After this part of the lecture and additional reading, you should be able to ...

1. ...understand the fundamentals behind generator scheduling;
2. ...perform simple economic dispatch analysis analytically and by computer software; and
3. ...perform simple unit commitment analysis by computer software.

1 Outline

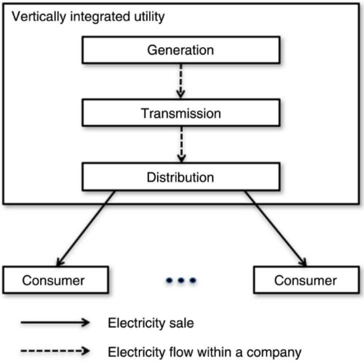
1 Reminder on power-frequency control



Generator scheduling?

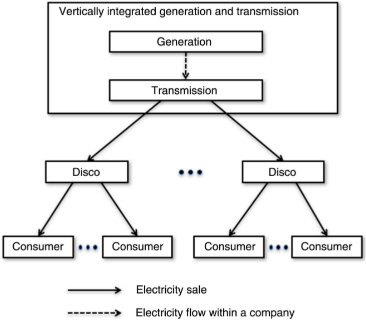
1 Traditional model of electricity supply

- A monopoly (vertically integrated utility)
- Usually, government entities or be subject to oversight by a government department



1 Variant on the traditional model of electricity supply

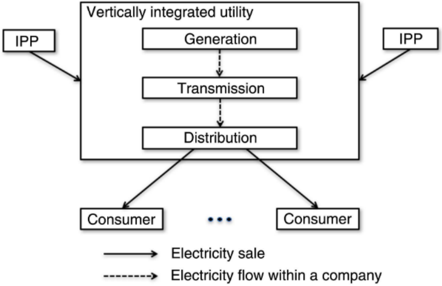
- The vertically integrated utility is split in two parts
- Generation and transmission has monopoly over entire area (often government owned)
- Multiple distribution companies have monopoly over their areas (often privately owned)



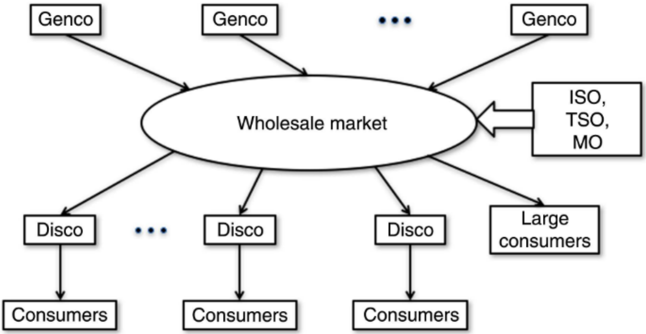
Monopolies tend to be inefficient because they do not have to compete with others in order to survive

1 Vertically integrated utility with independent power producers (IPPs)

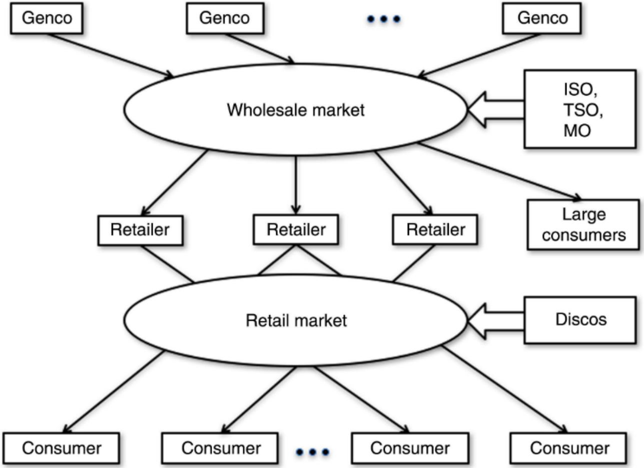
- Introduces a degree of competition at the generation level



1 Wholesale electricity market structure



1 Wholesale electricity market structure with retail competition



1 Stakeholders

- **Genco - Generating company**
  The Genco owns production assets (from single generator to a portfolio), whose generation is offered through the electricity market.
- **Retailer**
  The Retailer buys electricity en gros from the wholesale electricity market, to then be sold to the end-consumers.
- **Consumers (large and small)**
  Those eventually use the electricity for any purpose (from watching TV to heating to industrial production processes). There is a difference between small and large consumers, since the latter ones may be allowed to directly participate in the wholesale electricity market.

1 Stakeholders

- **Regulator**
  The regulator is responsible for the market design and its specific rules. It also monitors the market in order to spot misbehavior in electricity markets (collusion, abuse of market power, etc.). Exs: The Danish Energy Regulatory Authority DERA, CRE in France, Ofgem in the UK, CERA in Cyprus etc.
- **The Market Operator**
  The Market Operator organizes and operates the market place. This may include the definition of bid products and bid forms, set up and maintenance of the trading platform, daily matching of supply and demand offers, etc. Ex: Nord Pool, APX, EEX, PowerNext, TSO in Cyprus, etc.

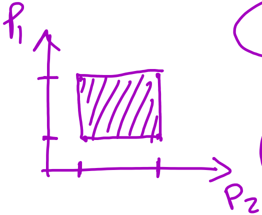
1 Generation scheduling

- **Economic Dispatch (ED)**: What is the most ***economic*** way to supply a load P_D^{total} using N_{gen} generators? All generators are connected and we can only select the power output.
- **Unit Commitment (UC)**: What is the most ***economic*** way to supply a load P_D^{total} using N_{gen} generators? We can select the output as well as which generators are connected.
- **Market clearing procedures**: For a given set of offers by producers and bids by consumers, a market-clearing procedure is an algorithm used by the market operator to determine (i) accepted offers, (ii) accepted bids, and (iii) clearing prices.

2 Outline

✍ What is the most ***economic*** way to supply a load P_D^{total} using N_{gen} generators? The cost of generating power P_{Gi} from the i-th generator is given by $C_i(P_{Gi})$.

Mathematically, this is an **optimization problem**:



$$\underset{P_{Gi}}{\text{minimize}} \quad C(P_G) = \sum_{i=1}^{N_{gen}} C_i(P_{Gi}) \tag{2.1a}$$

subject to

$$\sum_{i=1}^{N_{gen}} P_{Gi} - P_D^{total} - P_{loss} = 0, \tag{2.1b}$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \quad i = 1, \ldots, N_{gen} \tag{2.1c}$$

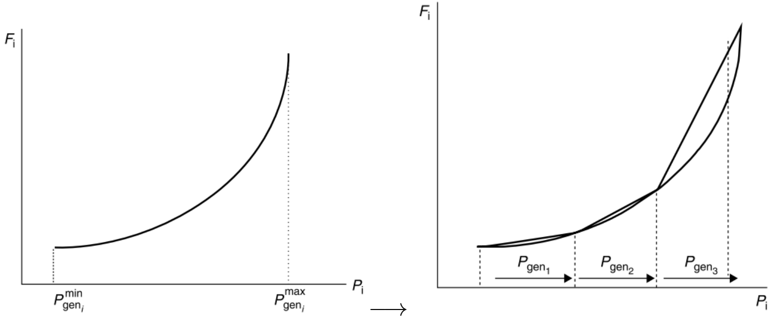
2.1 Economic Dispatch problem

- Usually, a quadratic cost is used:

$$C_i(P_{Gi}) = C_{0i} + a_i P_{Gi} + \frac{1}{2} b_i P_{Gi}^2$$

where C_{0i} is the fixed cost$(€/\mathrm{h})$, with the parameters $a_i(€/\mathrm{MWh})$ and b_i $(€/(\mathrm{MW})^2\ \mathrm{h})$ characterizing the variable cost component depending on the generation level $(P_{\mathrm{G}i})$

- However, this makes the problem **quadratic**, which is harder to solve. So, for large-scale problems, *piecewise-linear functions* are derived.



If we ignore all limits and losses, the problem becomes:

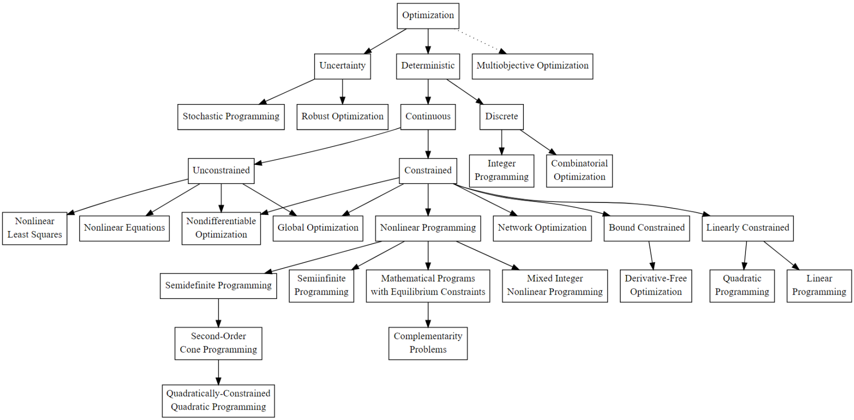
$$\underset{P_{Gi}}{\text{minimize}} \quad \sum_{i=1}^{N_{gen}} C_i(P_{gi}) \tag{2.2a}$$

subject to

$$\sum_{i=1}^{N_{gen}} P_{Gi} - P_D^{total} = 0 \tag{2.2b}$$

This is a Quadratic Problem (QP) and can be solved be using a *Lagrange function*.

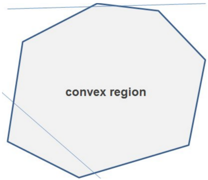
2.1 Side-note: Optimization Taxonomy



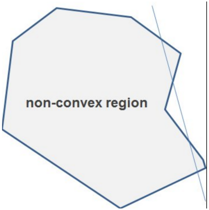
Source: https://neos-guide.org/content/optimization-taxonomy

2.1 Side-note: Optimization

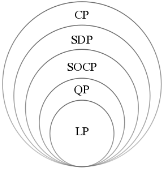
- **Convex**: All of the constraints are convex functions, and the objective is a convex function if minimizing, or a concave function if maximizing. Only one optimum exists which is also a global optimum. Can be solved efficiently up to very large size (hundreds of thousands of variables and constraints).



- **Non-convex**: Multiple local optimum. Hard to find the global optimum. It can take time exponential in the number of variables and constraints.



- A hierarchy of convex optimization problems. (LP: linear program, QP: quadratic program, SOCP second-order cone program, SDP: semidefinite program, CP: cone program.)



2.1 Basic ED problem

- The Langrange function is formulated by combining the objective function and the constraints:

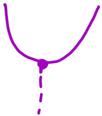
$$\mathcal{L}(P_G,\lambda)=\sum_{i=1}^{N_{gen}} C_i(P_{gi})-\lambda\left(\sum_{i=1}^{N_{gen}} P_{Gi}-P_D^{total}\right)$$

where λ is called the Lagrange multiplier.

- The first-order necessary optimality conditions that minimize the function and give the stationary points are

$$\frac{\partial \mathcal{L}(\cdot)}{\partial P_{\mathrm{G}i}}=IC_i\left(P_{\mathrm{G}i}\right)-\lambda=0, \quad i=1,\ldots,N_{gen}$$

$$\frac{\partial \mathcal{L}(\cdot)}{\partial \lambda}=-\sum_{i=1}^{N_{gen}} P_{\mathrm{G}i}+P_{\mathrm{D}}^{\mathrm{total}}=0$$



where the function $IC_i\left(P_{\mathrm{G}i}\right)$ is the **incremental cost** of unit i

$$IC_i\left(P_{\mathrm{Gi}}\right)=\frac{\mathrm{d}C_i\left(P_{\mathrm{G}i}\right)}{\mathrm{d}P_{\mathrm{G}i}}$$

2.1 Basic ED problem

- Solving the $N_{gen}+1$ equations coming from the optimality conditions will give the solution $[P_{G1},\dots,P_{GN_{gen}},\lambda]$, that minimizes the objective.
- The equations further imply that under ED, all units must operate at identical incremental costs equal to the Lagrange multiplier λ.
- It is interesting to observe that the common incremental cost λ also coincides with the **system marginal cost**, that is, the sensitivity of the total cost with respect to the system demand,

$$\lambda = \frac{\mathrm{d}C\left(P_{\mathrm{G}}\right)}{\mathrm{d}P_{\mathrm{D}}^{\text{total}}}$$

  also referred to as **the cost of the "last" MW added to the demand**.
- A change in the cost is connected to a change in the demand through:

$$\mathrm{d}C\left(P_{\mathrm{G}}\right)=\sum_{i=1}^{n}\mathrm{d}C_i\left(P_{\mathrm{G}i}\right)=\sum_{i=1}^{n}IC_i\left(P_{\mathrm{Gi}}\right)\mathrm{d}P_{\mathrm{G}i}=\sum_{i=1}^{n}\lambda\mathrm{d}P_{\mathrm{G}i}=\lambda\mathrm{d}P_{\mathrm{D}}^{\text{total}}$$

If we add the generator limits to the problem, the Lagrange function[1] then becomes:

$$\mathcal{L}\left(P_{\mathrm{G}}, \lambda\right) = \sum_{i=1}^{N_{gen}} C_i\left(P_{\mathrm{G}i}\right) - \lambda\left(\sum_{i=1}^{N_{gen}} P_{\mathrm{G}i} - P_{\mathrm{D}}^{\mathrm{total}}\right)$$

$$- \sum_{i=1}^{N_{gen}} \mu_i^{\max}\left(P_{\mathrm{G}i} - P_{\mathrm{G}i}^{\max}\right)$$

$$- \sum_{i=1}^{N_{gen}} \mu_i^{\min}\left(P_{\mathrm{G}i} - P_{\mathrm{G}i}^{\min}\right)$$

$P_{Gi}^{min} < P_{Gi} < P_{Gi}^{max}$

$P_{Gi} - P_{Gi}^{min} > 0$

$P_{Gi} - P_{Gi}^{max} < 0$

[1] Check A. Gómez-Expósito, A. J. Conejo, and C. A. Canizares, Electric Energy Systems Analysis and Operation, 2nd edition, CRC Press, 2018.

2.1 ED problem with generator limits

The first-order necessary optimality conditions become

$$\frac{\partial \mathcal{L}(\cdot)}{\partial P_{\mathrm{G}i}} = IC_i\left(P_{\mathrm{G}i}\right) - \lambda - \mu_i^{\max} - \mu_i^{\min} = 0; \quad i = 1, \ldots, N_{gen}$$

$$\frac{\partial \mathcal{L}(\cdot)}{\partial \lambda} = -\sum_{i=1}^{N_{gen}} P_{\mathrm{G}i} + P_{\mathrm{D}}^{\text{total}} = 0$$

including the complementarity slackness conditions

$$\begin{aligned}
\mu_i^{\max} &\leq 0 && \text{if } P_{\mathrm{G}i} = P_{\mathrm{G}i}^{\max} \\
\mu_i^{\max} &= 0 && \text{if } P_{\mathrm{G}i} < P_{\mathrm{G}i}^{\max} \\
\mu_i^{\min} &\geq 0 && \text{if } P_{\mathrm{G}i} = P_{\mathrm{G}i}^{\min} \\
\mu_i^{\min} &= 0 && \text{if } P_{\mathrm{G}i} > P_{\mathrm{G}i}^{\min}
\end{aligned}$$

If generation limits are imposed, the equal marginal cost condition of the basic ED **is no longer valid** It is replaced instead by the criterion below:

$$\begin{aligned}
IC_i\left(P_{\mathrm{G}i}\right) &= \lambda + \mu_i^{\min} \geq \lambda && \text{if } P_{\mathrm{G}i} = P_{\mathrm{G}i}^{\min} \\
IC_i\left(P_{\mathrm{G}i}\right) &= \lambda && \text{if } P_{\mathrm{G}i}^{\min} < P_{\mathrm{G}i} < P_{\mathrm{G}i}^{\max} \\
IC_i\left(P_{\mathrm{G}i}\right) &= \lambda + \mu_i^{\max} \leq \lambda && \text{if } P_{\mathrm{G}i} = P_{\mathrm{G}i}^{\max}
\end{aligned} \tag{2.3}$$

2.1 ED problem with generator limits

- The Lagrange multiplier λ can still be interpreted as the system marginal cost, in other words, the sensitivity of the system cost with respect to the system demand.
  - Units operating within its upper and lower limits exhibit identical incremental costs **equal** to λ.
  - Units operating at their maximum capacity have incremental costs **lower than or equal** to λ.
  - Units operating at their minimum power output have incremental costs **greater than or equal** to λ.
- If the cost functions are convex, the solution of the ED with generation limits is unique and easy to compute numerically.
- An analytic solution **is not easily obtained** because it is necessary to consider all possible combinations of units either operating at their respective limits or not, a difficult ***combinatorial problem*** in general.
- Need to use using mathematical programming solvers (e.g., CPLEX, Pyomo, etc.) or specific procedures can be used to efficiently solve the generation-constrained ED problem (e.g., λ-iteration algorithm)

Step 1. The multiplier λ is approximated by $\lambda^{(\nu)}$.

Step 2. The unit generation levels are computed so that the optimality conditions (2.3) are satisfied, that is,

$$\begin{aligned} &\text{if} \quad IC_i\left(P_{\mathrm{G}i}^{\min}\right) \geq \lambda^{(\nu)}, \quad \text{then } P_{\mathrm{G}i} = P_{\mathrm{G}i}^{\min} \\ &\text{else if } IC_i\left(P_{\mathrm{G}i}^{\max}\right) \leq \lambda^{(\nu)}, \quad \text{then } P_{\mathrm{G}i} = P_{\mathrm{G}i}^{\max} \\ &\text{otherwise, compute } P_{\mathrm{G}i} \text{ so that } IC_i\left(P_{\mathrm{G}i}\right) = \lambda^{(\nu)} \end{aligned}$$

The total generation level is calculated adding the generation levels of all units, and the balance of generation and demand is checked. If the balance is satisfied within a given tolerance, go to *Step 4*. Otherwise, go to *Step 3*.

Step 3. To update λ, a bisection rule is used, $\lambda^{(\nu+1)} = \left[\lambda^{(\nu)} + \lambda^{(\nu-1)}\right]/2$, with the previous values of λ corresponding to a generation surplus and to a generation deficit, respectively. Go to *Step 2*.

Step 4. Done.

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- Losses can be incorporated into the ED through the following modified power balance equation:

$$\sum_{i=1}^{N_{gen}} P_{\mathrm{G}i} - P_{\mathrm{D}}^{\mathrm{total}} - P_{\mathrm{loss}}\left(P_{\mathrm{G}}, P_{\mathrm{D}}\right) = 0$$

- The losses slightly increase the demand (usually, 3-5%).
- Considering the power balance equation with losses, the Lagrangian function becomes

$$\mathcal{L}\left(P_{\mathrm{G}}, \lambda\right) = \sum_{i=1}^{n} C_i\left(P_{\mathrm{G}i}\right) - \lambda\left[\sum_{i=1}^{n} P_{\mathrm{G}i} - P_{\mathrm{D}}^{\mathrm{total}} - P_{\mathrm{loss}}\left(P_{\mathrm{G}}, P_{\mathrm{D}}\right)\right]$$

- The first-order necessary conditions are

$$\frac{\partial \mathcal{L}(\cdot)}{\partial P_{\mathrm{G}i}} = IC_i\left(P_{\mathrm{G}i}\right) - \lambda\left(1 - \frac{\partial P_{\mathrm{loss}}}{\partial P_{\mathrm{G}i}}\right) = 0, \quad i = 1, \ldots, N_{gen}$$

$$\frac{\partial \mathcal{L}(\cdot)}{\partial \lambda} = -\sum_{i=1}^{N_{gen}} P_{\mathrm{G}i} + P_{\mathrm{D}}^{\mathrm{total}} + P_{\mathrm{loss}}\left(P_{\mathrm{G}}, P_{\mathrm{D}}\right) = 0$$

A convenient way to represent the power flow losses is through a simplified loss formula[2]:

$$P_{loss} = a_1^{loss} P_{G1}^2 + \ldots + a_{N_{gen}}^{loss} P_{GN_{gen}}^2$$

$$\frac{\partial P_{loss}}{\partial P_{G1}} = 2a_1^{loss} \cdot P_{G1}$$

However, in the general case, the conditions are no longer linear and cannot be easily computed. An iterative method is needed:

Step 1. Pick a set of starting values for $P_{G1}, \ldots, P_{N_{gen}}$ that sum to the load.

Step 2. Calculate the incremental losses $\partial P_{\text{loss}} / \partial P_{Gi}$ as well as the total losses P_{loss}. The incremental losses and total losses will be considered constant until we return to *Step 2*.

Step 3. Calculate the value of λ that causes P_{Gi} to sum to the total load plus losses. This is now as simple since the equations are again linear.

Step 4. Compare the P_{Gi} from *Step 3* to the values used at the start of *Step 2*. If there is no significant change in any one of the values, go to *Step 5*; otherwise, go back to *Step 2*.

Step 5. Done.

[2] L. K. Kirchmayer, Economic Operation of Power Systems, Wiley & Sons, 1958

Until now, we ignored the limits on the transmission lines. How do we include them?

$$\underset{P_{Gi}}{\text{minimize}} \quad C(P_G) = \sum_{i=1}^{N_{gen}} C_i(P_{Gi}) \tag{2.4a}$$

subject to

$$\sum_{i=1}^{N_{gen}} P_{Gi} - P_D^{total} = 0, \tag{2.4b}$$

$$\text{power-flow equations,} \tag{2.4c}$$

Non-linear

$$\left| P_F \right| \leq P_F^{max}, \tag{2.4d}$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \quad i = 1, \ldots, N_{gen} \tag{2.4e}$$

Power-flow equations very ***non-linear***! The problem is NLP, very hard to solve! $\longrightarrow$ Use DC power flow!

2.3 Reminder: DC power flow equations - Assumptions and procedure

- DC power flow equations are simplified equations obtained after
  1. Neglecting reactive power flows in all branches
  2. Neglecting active power losses in all branches
  3. Assuming all voltage magnitudes equal to 1pu
  4. Angle difference between buses is small
- Hence, we assume
  1. $V_k = 1$ pu, $k = 1, \ldots, N$
  2. $G_{km} = 0$ for all $k = 1, \ldots, N$, $m = 1, \ldots, N$
  3. $B_{km} = -\frac{1}{X_{km}}$ for all $k = 1, \ldots, N$, $m = 1, \ldots, N$
  4. $a_{km} = 1$ (transformer ratio influences mainly reactive power flows)

2.3 Reminder: DC power flow equations - Approximate equations

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- Linearisation of active power flow from node k to node m

$$
\begin{aligned}
P_{km}^{\text{DC}} &= \frac{\partial P_{km}}{\partial \theta_k}\theta_k + \frac{\partial P_{km}}{\partial \theta_m}\theta_m \\
&= \frac{\cos(\theta_{km})}{X_{km}}\theta_k + \frac{-\cos(\theta_{km})}{X_{km}}\theta_m = \frac{\cos(\theta_{km})}{X_{km}}\theta_{km} \\
&\approx \frac{1}{X_{km}}\theta_{km} \quad \text{(approximation valid for small phase angle differences)}
\end{aligned}
$$

- DC power flow can be written in matrix form as follows

$$\boldsymbol{P}' = \boldsymbol{A}'\boldsymbol{\theta},$$

  where
  - $\boldsymbol{P}'$ is vector of net active power injections (generation-load at each bus)
  - $\boldsymbol{\theta}$ is vector of voltage angles
  - $\boldsymbol{A}'$ is nodal admittance matrix with elements

$$A_{km} = -X_{km}^{-1}, \qquad A_{kk} = \sum_{m \in \mathcal{N}_k} X_{km}^{-1}$$

- To make system of equations solvable, we (arbitrarily) chose one bus as angle reference and remove row and column associated with that bus from $\boldsymbol{A}'$; we shall call that reduced matrix $\boldsymbol{A}$

Using the above formulation, we can define the Network-constrained ED:

$$\underset{P_{Gi},\,\theta}{\text{minimize}} \quad C(P_G) = \sum_{i=1}^{N_{gen}} C_i(P_{Gi}) \tag{2.5a}$$

subject to

$$\sum_{i=1}^{N_{gen}} P_{Gi} - P_D^{total} = 0, \quad - P_{losses} = 0 \tag{2.5b}$$

$$\boldsymbol{P} = \boldsymbol{A}\boldsymbol{\theta}, \tag{2.5c}$$

$$\left| \frac{1}{X_{km}} \theta_{km} \right| \leq P_{km}^{max} \quad \forall k, m \in \mathcal{N}_k, \tag{2.5d}$$

$$P_{Gi}^{min} \leq P_{Gi} \leq P_{Gi}^{max} \quad i = 1, \ldots, N_{gen} \tag{2.5e}$$

3 Outline

ED

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- In ED all the generators are considered to be **already** online and there are no constraints (or cost) in start-up or shut-down.
- In UC there are N_{gen} generators **available** but not necessarily online. So, the UC problem asks:

> ✍ Given that there are a number of subsets of the complete set of N_{gen} generating units that would satisfy the expected demand, which of these subsets should be used in order to provide the minimum operating cost?

- The ON/OFF decision variables convert the problem into a *Mixed-Integer* optimization problem, which is hard to solve.

P

If we assume that u_i is a **binary variable** that shows if the unit is connected or not (0 $\rightarrow$ not connected, 1 $\rightarrow$ connected). The ED problem can be converted to a UC problem as:

$$\underset{P_{Gi},\, u_i}{\text{minimize}} \quad \sum_{i=1}^{N_{gen}} C_i(u_i, P_{Gi}) \tag{3.1a}$$

subject to

$$\sum_{i=1}^{N_{gen}} P_{Gi} - P_D^{total} = 0, \tag{3.1b}$$

$$u_i P_{Gi}^{min} \leq P_{Gi} \leq u_i P_{Gi}^{max} \quad \forall i, \tag{3.1c}$$

$$u_i \in \{0, 1\} \quad \forall i \tag{3.1d}$$

Where the cost function is modified as:

$$C_i(u_i, P_{Gi}) = u_i C_{0i} + a_i P_{Gi} + \frac{1}{2} b_i P_{Gi}^2$$

If we are scheduling for multiple consecutive hours (T), the problem is reformulated as:

$$\underset{P_{Git},\, u_{it}}{\text{minimize}} \quad \left(\sum_{t=1}^{T} \sum_{i=1}^{N_{gen}} C_i(u_{it}, P_{Git}) + C_{it}^{SU} \right) \tag{3.2a}$$

subject to

$$\sum_{i=1}^{N_{gen}} P_{Git} - P_{Dt}^{total} = 0 \quad \forall t, \tag{3.2b}$$

$$u_{it} P_{Git}^{min} \leq P_{Git} \leq u_{it} P_{Git}^{max} \quad \forall i, \forall t, \tag{3.2c}$$

$$C_{it}^{\text{SU}} \geq C_{i}^{\text{SU}} (u_{it} - u_{i,t-1}) \quad \forall i, \forall t, \tag{3.2d}$$

$$C_{it}^{\text{SU}} \geq 0 \quad \forall i, \forall t, \tag{3.2e}$$

$$P_{\text{G}i,t-1} - P_{\text{G}it} \leq R_{\text{G}i}^{\text{down}} \quad \forall i, \forall t, \tag{3.2f}$$

$$u_{it} \in \{0, 1\} \quad \forall i, \forall t \tag{3.2g}$$

$P_{Git} - P_{Gi,t-1} \leq R_{Gi}^{up}$

3.1 Spinning reserve

- Spinning reserve is the term used to describe the total amount of generation available from all units synchronized (i.e., spinning) on the system, minus the present load and losses being supplied. Thus, the constraint related to the power balance becomes

$$\sum_{i=1}^{N_{gen}} u_{it} P_{\mathrm{G}i}^{\max} \geq P_{\mathrm{D}t}^{total} + P_{\mathrm{R}t} \; \forall t$$

- Spinning reserve must be carried so that the loss of one or more units does not cause too far a drop in system frequency.
- Beyond spinning reserve, the UC problem may involve various classes of ”scheduled reserves” or ”off-line” reserves.

Assume:

- We must establish a loading pattern for M periods.
- We have N_{gen} units to commit and dispatch.
- The M load levels and operating limits on the N_{gen} units are such that any one unit can supply the individual loads and that any combination of units can also supply the loads.

If we try by enumeration (brute force), the number of combinations we need to try **each hour** is:

$$C(N_{gen}, 1) + C(N_{gen}, 2) + \cdots + C(N_{gen}, N_{gen} - 1) + C(N_{gen}, N_{gen}) = 2^{N_{gen}} - 1$$

where $C(N_{gen}, j)$ is the combination of N_{gen} items taken j at a time.

That is:

$$C\left(N_{\text{gen}}, j\right) = \left[\frac{N_{\text{gen}}!}{\left(N_{\text{gen}} - j\right)! j!}\right]$$

$$j! = 1 \times 2 \times 3 \times \cdots \times j$$

For the total period of M intervals, the maximum number of possible combinations is $\left(2^{N_{gen}} - 1\right)^M$.

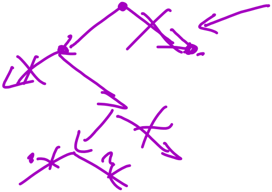
For a 24-h period (e.g., 24 one-hour intervals) and consider systems with 5, 10, 20, and 40 units. The value of $\left(2^{N_{gen}} - 1\right)^{24}$ becomes the following.

N_{gen}	$\left(2^{N_{gen}} - 1\right)^{24}$
5	6.2×10^{35}
10	1.73×10^{72}
20	3.12×10^{144}
40	(Too big)

3.1 UC problem solution algorithms

To reduce the complexity, several methods are used. Some of the most popular:

- Priority-list schemes,
- Dynamic programming (DP),
- Lagrange relaxation (LR).
- Mixed integer linear programming (MILP) $\rightarrow$ See example in Pyomo.



4 Outline

4 Slides based on

Cyprus University of Technology

[1] Kirschen, D., & Strbac, G. (2018). Fundamentals of Power System Economics. In Wiley Online Library. John Wiley & Sons, Ltd.

[2] Wood, A. J., Wollenberg, B. F., & Sheble, G. B. (2014). Power generation, operation, and control (3rd ed.). Wiley-IEEE Press.

[3] Gómez-Expósito, A., Conejo, A. J., & Cañizares, C. A. (2018). Electric Energy Systems Analysis and Operation. CRC Press.