A Data-Driven Two-Stage Distributionally Robust Planning Tool for Sustainable Microgrids

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***Abstract*—This paper presents a data-driven two-stage distributionally robust planning tool for sustainable microgrids under the uncertainty of load and power generation of renewable energy sources (RES) during the planning horizon. In the proposed two-stage planning tool, the first-stage investment variables are considered as *here-and-now* decisions and the second-stage operation variables are considered as *wait-and-see* decisions. In practice, it is hard to obtain the true probability distribution of the uncertain parameters. Therefore, a Wasserstein metric-based ambiguity set is presented in this paper to characterize the uncertainty of load and power generation of RES without any presumption on their true probability distributions. In the proposed data-driven ambiguity set, the empirical distributions of historical load and power generation of RES are considered as the center of the Wasserstein ball. Since the proposed distributionally robust planning tool is intractable and it cannot be solved directly, duality theory is used to come up with a tractable mixed-integer linear (MILP) counterpart. The proposed model is tested on a 33-bus distribution network and its effectiveness is showcased under different conditions.**

NOMENCLATURE

Indices

b	Index of buses where b' and b'' represent indices for buses before and after bus b, respectively.
d	Index of loads.
g	Index of generators.
t	Index of hours.

Parameters

cp	Cost of penalization for non-supplied load ($/kWh).
e_t^b/e_t^s	Buying/selling price of electricity from/to the bulk upstream network at hour t ($/kWh).
f_d	Power factor of load d.
$i_{bb''}$	Annualized investment/reinforcement cost of line connecting buses (b,b'') ($).
i_g	Annualized investment cost of generator g ($).
o_{gt}	Operation cost of generator g at hour t ($/kWh).
$\bar{p}_{dt}$	Nominal estimate of $\tilde{p}_{dt}$ ($/kWh).
$p_{b'b}^{\max}$	Maximum active power flow from bus b' to bus b (kW).
$p_g^{\max}$	Maximum active power of generator g (kW).
$\bar{p}_{gt}^{\max}$	Nominal estimate of $\tilde{p}_{gt}^{\max}$ (kW).
$q_{b'b}^{\max}$	Maximum reactive power flow from bus b' to bus b (kVAr).
$q_g^{\max}$	Maximum reactive power of generator g (kVAr).
$r_{b'b}$	Resistance of line connecting buses (b',b) (ohm).
$v^{\max}$	Maximum permitted voltage amplitude (V).
$v^{\min}$	Minimum permitted voltage amplitude (V).
$x_{b'b}$	Reactance of line connecting buses (b',b) (ohm).
$\alpha_{bb''y}^0$	Initial connection status of buses (b,b'') (i.e., 1/0: connected/disconnected).

Sets

Ω^B	Set of buses where Ω^{B_b} indicates set of buses after and connected to bus b.
Ω^D	Set of loads where Ω^{D_b} indicates set of loads connected to bus b.
Ω^L	Set of lines connecting buses.
Ω^M	Set of micro-turbine generators where Ω^{M_b} indicates set of micro-turbine generators connected to bus b.
Ω^R	Set of renewable generators where Ω^{R_b} indicates set of renewable generators connected to bus b.
Ω^T	Set of hours.

Variables

$\tilde{p}_{dt}$	Uncertain load d at hour t (kW).
$p_{b'bt}$	Active power flow from bus b' to bus b at hour t (kW).
p_t^b/p_t^s	Active power bought/sold from/to the bulk upstream network at hour t (kW).
p_{gt}	Active power generation of generator g at hour t (kW).
$\tilde{p}_{gt}^{\max}$	Uncertain maximum active power generation of generator g at hour t (kW).
$q_{b'bt}$	Reactive power flow from bus b' to bus b at hour t (kVAr).
q_{gt}	Reactive power generation of generator g at hour t (kVAr).
u_{dt}	Not-supplied value of load d at hour t (kW).
v_{bt}	Voltage amplitude of bus b at hour t (V).
$\alpha_{bb''}$	Investment/reinforcement status of line connecting buses (b,b'') (i.e., 1/0: built/non-built).
α_g	Investment status of generator g (i.e., 1/0: built/non-built).

I. INTRODUCTION

The concept of microgrid (MG) has been firstly presented in [1] as a solution to address different challenges of integrating distributed energy resources into the power system. In general, MG refers to a low-voltage electrical network with small-scale producers and consumers that operate as a self-sufficient controllable system. Although fossil-fuel-based generation technologies with competitive installation costs have

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been the predominant choice to supply electricity in remote areas, the proven techno-economic feasibility and practicality of sustainable generation technologies (e.g., wind turbines and solar panels) have made them a priority in MGs [2]. Therefore, efficient MG investment/reinforcement planning (MIRP) models are required for optimal investment/reinforcement in different generation technologies, e.g., renewable energy sources (RES) and fossil-fuel ones, as well as distribution facilities.

In practice, MGs are subject to various uncertainties (i.e., load and power generation of RES) that affect the operational feasibility of any investment plan. Accordingly, to characterize these uncertainties and solve the MIRP problem, different non-deterministic techniques have been introduced in the literature, including robust optimization (RO) [3], [4] and stochastic optimization (SO) [5], [6]. RO finds a solution that is optimal under the worst-case realization of uncertain parameters while SO finds a solution that is optimal on average for all scenarios considered in the problem. On the one hand, RO needs less historical data than SO to characterize uncertain parameters [7]. On the other hand, optimal solutions of RO-based MIRP models are more conservative than the optimal solutions of SO-based ones. To remedy the aforementioned shortcomings of RO and SO, and to combine their benefits, a data-driven two-stage distributionally robust MIRP (DR-MIRP) model is introduced in this paper under the uncertainty of loads and power generations of RESs. The optimal solution in the DR-MIRP model is obtained under the worst-case probability distributions of the uncertain parameters, rather than their worst-case realizations similarly to [8], where the investment variables are considered as *here-and-now* decisions and operation variables are considered as *wait-and-see* decisions.

The main contributions of this paper can be summarized as follows: (i) A data-driven two-stage distributionally robust model is introduced for investment/reinforcement planning in sustainable MGs under the uncertainty of loads and power generations of RESs during a single-year planning horizon. To the best of the authors' knowledge, there is no similar data-driven distributionally robust MG planning model in the literature; (ii) The uncertainty of loads and power generations of RESs is characterized by a data-driven Wasserstein ambiguity set where there is no presumption on true probability distributions of uncertain parameters. Also, empirical distributions pertaining to previous observations of uncertain parameters are used to construct the data-driven Wasserstein-based ambiguity set; (iii) A tractable mixed-integer linear programming (MILP) counterpart for the proposed intractable DR-MIRP is presented using the duality theory. Also, the conservatism of the optimal solution can be adjusted by means of the confidence level of the Wasserstein ambiguity set.

The rest of this paper is organized as follows. In Section II, a deterministic formulation is presented for MG planning. In Section III, first, the ambiguity set is introduced, second, the distributionally robust formulation is presented for MG planning, and then, its MILP counterpart is obtained by using the duality theory. In Section IV, the proposed DR-MIRP model is tested on a 33-bus distribution network under different conditions. Finally, Section V concludes the paper.

II. Deterministic Formulation

The deterministic MIRP (D-MIRP) model is proposed in this section where future profiles of loads and power generations of RESs is modeled by a representative operating day:

$$\min \sum_{(b,b'')\in\Omega^L}\left(\frac{i_{bb''}\cdot\alpha_{bb''}}{365}\right)+\sum_{g\in\{\Omega^M,\Omega^R\}}\left(\frac{i_g\cdot\alpha_g}{365}\right) + \sum_{g\in\{\Omega^M,\Omega^R\}}\sum_{t\in\Omega^T}(o_{gt}\cdot p_{gt})+\sum_{d\in\Omega^D}\sum_{t\in\Omega^T}(cp\cdot u_{dt}) + \sum_{t\in\Omega^T}\left(e^b_t\cdot p^b_t-e^s_t\cdot p^s_t\right) \tag{1a}$$

s.t.

$$p_{b'bt}+\sum_{g\in\{\Omega^{M_b},\Omega^{R_b}\}}p_{gt}=\sum_{b''\in\Omega^{B_b}}p_{bb''t}+ \sum_{d\in\Omega^{D_b}}(\bar{p}_{dt}-u_{dt}) \quad b\in\Omega^B,t\in\Omega^T \tag{1b}$$

$$q_{b'bt}+\sum_{g\in\Omega^{M_b}}q_{gt}=\sum_{b''\in\Omega^{B_b}}q_{bb''t}+ \sum_{d\in\Omega^{D_b}}\tan(\arccos(f_d))\cdot(\bar{p}_{dt}-u_{dt}) \quad b\in\Omega^B,t\in\Omega^T \tag{1c}$$

$$v_{b't}-v_{bt}=(r_{b'b}\cdot p_{b'bt}+x_{b'b}\cdot q_{b'bt}) \quad b\in\Omega^B,t\in\Omega^T \tag{1d}$$

$$-p^{\max}_{bb''}\cdot\left(\alpha^0_{bb''}+\alpha_{bb''}\right)\le p_{bb''t}\le p^{\max}_{bb''}\cdot\left(\alpha^0_{bb''}+\alpha_{bb''}\right) \quad (b,b'')\in\Omega^L,t\in\Omega^T \tag{1e}$$

$$-q^{\max}_{bb''}\cdot\left(\alpha^0_{bb''}+\alpha_{bb''}\right)\le q_{bb''t}\le q^{\max}_{bb''}\cdot\left(\alpha^0_{bb''}+\alpha_{bb''}\right) \quad (b,b'')\in\Omega^L,t\in\Omega^T \tag{1f}$$

$$p_{01t}=p^b_t-p^s_t \quad t\in\Omega^T \tag{1g}$$

$$0\le p^b_t \quad ; \quad 0\le p^s_t \quad t\in\Omega^T \tag{1h}$$

$$0\le p_{gt}\le p^{\max}_g\cdot\alpha_g \quad g\in\Omega^M,t\in\Omega^T \tag{1i}$$

$$q^{\min}_g\cdot\alpha_g\le q_{gt}\le q^{\max}_g\cdot\alpha_g \quad g\in\Omega^M,t\in\Omega^T \tag{1j}$$

$$0\le p_{gt}\le \bar{p}^{\max}_{gt}\cdot\alpha_g \quad g\in\Omega^R,t\in\Omega^T \tag{1k}$$

$$0\le u_{dt} \quad d\in\Omega^D,t\in\Omega^T \tag{1l}$$

$$v^{\min}\le v_{bt}\le v^{\max} \quad b\in\Omega^B,t\in\Omega^T \tag{1m}$$

$$v_{1t}=1 \quad t\in\Omega^T \tag{1n}$$

In (1a), the objective function calculates 1) the total costs of investment/reinforcement in distribution and generation facilities, 2) the total operation costs of micro-tribune and renewable generators, 3) the total cost of not-supplied load, and 4) the total operation costs of buying/selling electricity from/to the bulk upstream network during the entire planning horizon. Note that maintenance costs are included by aggregating investment and maintenance costs for all investment candidates. In this paper, the linearized version of the DistFlow model is used for the power flow equations [9], [10]. Also, a linearized approximation of the quadratic power flow limits is considered [11]. Therefore, constraints (1b) and (1c) ensure active and reactive power balance at each bus, respectively. In addition, constraint (1d) calculates the difference of voltage amplitudes between two neighbor connected buses. Constraints (1e) and (1f) limit the active and reactive power flows between two neighbor connected buses, respectively. Constraint (1g) stands for bought/sold electricity from/to the bulk upstream network where constraint (1h) represents the non-negativity of modeling variables for bought/soled electricity from/to the bulk upstream network. Constraints (1i) and (1j) bound active and reactive power generations of micro-turbine generators, respectively. Moreover, constraint (1k) bounds active power generations of renewable generators. Without loss of generality, a unity power factor is considered for all renewable generators [12]. Constraint (1l) represents the non-negativity of not-supplied loads. Constraint (1m) bounds the allowed variation interval of the voltage amplitude at each bus where constraint (1n) fixes the voltage amplitude of the bus that connects the MG to the bulk upstream network on one. For the sake of brevity, the proposed D-MIRP model in (1a)-(1n) can be presented in compact matrix form as follows:

$$\min_{x} \quad c^{\intercal} \cdot x + \sum_{t \in \Omega^T} S(x, \tilde{\eta}_t) \tag{2}$$

where $S(x, \tilde{\eta}) = \min_{y_t} \{d^{\intercal} \cdot y_t | E(x) + F \cdot y_t \geq G(x) \cdot \tilde{\eta}_t\}$ and $\tilde{\eta}_t$ represents the vector of uncertain parameters (i.e., $\tilde{p}_{dt}\ \forall d \in \Omega^d, \forall t \in \Omega^T$ and $\tilde{p}_{gt}^{\max}\ \forall g \in \Omega^R, \forall t \in \Omega^T$) and it is set on their nominal estimates $\bar{\eta}_t$ (i.e., $\bar{p}_{dt}\ \forall d \in \Omega^d, \forall t \in \Omega^T$ and $\bar{p}_{gt}^{\max}\ \forall g \in \Omega^R, \forall t \in \Omega^T$) in the D-MIRP model. Also, x and y_t represent the vector of binary investment variables and the vector of continuous operation variables at hour t, respectively. Vectors of costs and requirements, i.e., c and d, are indicated by lower-case letter while matrices of functions and coefficients, i.e., $E(x)$, F, and $G(x)$, are indicated by upper-case letters.

In summary, $c^{\intercal} \cdot x$ corresponds to the first and second terms of the objective function in (1a) while $\sum_{t \in \Omega^T} S(x, \tilde{\eta}_t)$ corresponds to the third, forth, and fifth terms of the objective function in (1a). Moreover, $E(x) + F \cdot y_t \geq G(x) \cdot \tilde{\eta}_t$ corresponds to all constraints (1b)-(1n).

III. Data-Driven Distributionally Robust Formulation

In this section, first, the ambiguity set is introduced, second, the distributionally robust model is presented, and then, its MILP counterpart is obtained by using the duality theory.

A. Data-Driven Ambiguity Set

To find the exact solution of a distributionally robust optimization, it is required to characterize exact probability distributions of uncertain parameters [13]. In practice, it is significantly hard to obtain the true probability distribution $\mathbb{P}$ (if not impossible) as there is limited historical data for uncertain parameters. However, the empirical distribution $\hat{\mathbb{P}}_{N_s}$ obtained from a sample set with N_s previous observations of uncertain parameters can be used to estimate the true probability distribution $\mathbb{P}$. The distance between the true distribution $\mathbb{P}$ and the empirical distribution $\hat{\mathbb{P}}_{N_s}$ can be reduced by increasing the number of historical data. In other words, $\hat{\mathbb{P}}_{N_s}$ tends toward $\mathbb{P}$ if $N_s \to \infty$ [14]. Accordingly, a data-driven approach based on previous observations of daily loads and power generations of RESs is introduced to construct the ambiguity set. In this paper, a Wasserstein metric-based ambiguity set as a type of discrepancy-based ambiguity sets is presented [15]. The main reasons are twofold. First, the proposed model only needs baseline estimates of uncertain parameters. Accordingly, it can be even used in areas with limited historical data (e.g., rural areas in countries with a low electrification rate). Second, the proposed model can be recast into a tractable MILP problem.

The Wasserstein metric $dist_W(\mathbb{P}_a, \mathbb{P}_b)$ as the distance between probability distributions $\mathbb{P}_a$ and $\mathbb{P}_b$ over the support space Ω can be defined as given below [14]:

$$dist_W(\mathbb{P}_a, \mathbb{P}_b) = \inf_{\Lambda} \{\mathbb{E}_{\Lambda}[\phi(x_a, x_b)] \,:\, x_a \sim \mathbb{P}_a \,,\, x_b \sim \mathbb{P}_b\} \tag{3}$$

where $\phi(x_a, x_b)$ represents a continuous distance between uncertain parameters x_a and x_b with probability distributions $\mathbb{P}_a$ and $\mathbb{P}_b$, respectively. Also, the infimum is calculated over all joint probability distributions Λ with marginal distributions $\mathbb{P}_a$ and $\mathbb{P}_b$.

Hence, the ambiguity set Θ_W can be presented as follows:

$$\Theta_W = \{\mathbb{P} \in \Xi(\Omega) \,:\, dist_W(\mathbb{P}, \hat{\mathbb{P}}_{N_s}) \leq \rho\} \tag{4}$$

where $\Xi(\Omega)$ stands for the set of all probability measures over the support set Ω. Moreover, $\hat{\mathbb{P}}_{N_s}$ and ρ represents the center and radius of the Wasserstein ball, respectively. The radius ρ is a function of the confidence level (i.e., γ), the diameter of the support set (i.e., D_Ω), and the number of samples (i.e., N_s) as given below [14]:

$$\rho = D_\Omega \sqrt{\frac{1}{2N_s} \ln\left(\frac{1}{1-\gamma}\right)} \tag{5}$$

In this paper, previous observations of daily patterns for uncertain loads and power generations of RESs are used as training samples to construct the ambiguity set.

B. Distributionally Robust Model

The DR-MIRP model in compact form can be presented as:

$$\min_{x} \quad c^{\intercal} \cdot x + \max_{\mathbb{P} \in \Theta_W} \mathbb{E}\left[\sum_{t \in \Omega^T} S(x, \tilde{\eta}_t)\right] \tag{6}$$

The proposed min-max-min formulation finds the optimal solution under the worst-case probability distribution belonging to the Wasserstein metric-based ambiguity set where investment variables x are here-and-know decisions and operation variables y_t at hour t are wait-and-see decisions. Clearly, this min-max-min formulation cannot be solved directly by available optimization packages, and consequently, a solvable MILP counterpart is presented in the sequel.

Given the sample set $\{\eta^1, ..., \eta^s, ..., \eta^{N_s}\}$ with $\eta^s = \{\eta_1^s, ..., \eta_t^s, ..., \eta_{24}^s\}$ for $s = 1, ..., N_s$ over the bounded support set $\Omega = \{\underline{\Omega}, \overline{\Omega}\}$, where $\underline{\Omega}$ and $\overline{\Omega}$ represents the lower and upper bounds of the support set, respectively, the inner max-min optimization problem in (6) can be rewritten as a minimization problem using the equivalence between the worst-case expectation and the optimal value of the generalized moment problem (i.e., Theorem 1 in [16]) as well as the strong linear programming duality (Theorem 6 in [16]).

Therefore, the min-max-min optimization problem in (6) can be recast into a single minimization problem as follows:

$$\min_{\Psi} \quad c^{\intercal} \cdot x + \rho \cdot \lambda + \frac{1}{N_s} \sum_{s=1}^{N_s} \sum_{t \in \Omega^T} \left(d^{\intercal} \cdot y_{st} \right) \tag{7a}$$

s.t.

$$\begin{aligned} & E(x) + F \cdot y_{st} \geq G(x) \cdot \tilde{\eta}_t^s \quad s = 1, .., N_s, t \in \Omega^T \\ & d^{\intercal} \cdot \sigma_{zt}^1 \leq \lambda \quad z = 1, ..., N_{\eta_t}, t \in \Omega^T \\ & d^{\intercal} \cdot \sigma_{zt}^2 \leq \lambda \quad z = 1, ..., N_{\eta_t}, t \in \Omega^T \\ & G(x) \cdot e_z \leq F \cdot \sigma_{zt}^1 \quad z = 1, ..., N_{\eta_t}, t \in \Omega^T \\ & -G(x) \cdot e_z \leq F \cdot \sigma_{zt}^2 \quad z = 1, ..., N_{\eta_t}, t \in \Omega^T \\ & \lambda \geq 0 \end{aligned} \tag{7b}$$

where $\Psi = \{\lambda, \sigma_{zt}^1, \sigma_{zt}^2, x, y_{st}\}$ represents the vector of optimization variables. Also, N_{η_t} denotes the number of uncertain parameters at hour t and e_z denotes a vector where its zth element is equal to one and its other elements are equal zero. The robust counterpart in (7a) and (7b) is a tractable MILP optimization problem, and it can be directly solved by available optimization solvers. Clearly, the computation time of the proposed MILP robust counterpart is a function of the number of training in-sample scenarios that are used to construct the ambiguity set.

IV. Case Studies

In this section, the proposed DR-MIRP model is tested on a 33-bus distribution network as illustrated in Fig. 1 [9]. All simulations are run on a server with 120 Intel Xeon processors and 102 GB of RAM. Also, the CPLEX solver in GAMS is used to solve MILP problems where the optimality gap is set to 10^{-2}. The nominal estimates of the representative operating day are obtained from the normalized patterns of loads and power generations of RESs in the electric reliability council of Texas (ERCOT) in 2014 where the representative operating day is obtained for the D-MIRP problem using the k-means clustering technique as depicted in Fig. 2 [17]. The prices of buying and selling electricity from and to the bulk upstream network are equal to $\$35$ and $\$10$, respectively. The annualized investment/reinforcement cost of lines is to 12000 \$/km. Also, the characteristics of micro-turbine and renewable generators are depicted in Table I [3]. In this study, 32 micro-turbine generators with a 1-MW capacity, 32 renewable generators with a 1-MW capacity, and 32 lines with a 400-kVA capacity connecting neighboring buses are considered as investment/reinforcement candidates. It is worthwhile to note that there in no investment/reinforcement candidate at the main bus connecting MG to the bulk upstream network.

To evaluate the performance of the proposed data-driven distributionally robust planning tool, DR-MIRP has been compared with deterministic (i.e., D-MIRP), and robust (i.e., R-MIRP) models as depicted in Table II. In this study, it is assumed that the support sets representing deviations of loads and power generations of RESs from their nominal estimates are bounded between $[\bar{p}_{dt}, 1.25 \cdot \bar{p}_{dt}]$ and $[0.85 \cdot \bar{p}_{gt}^{\max}, \bar{p}_{gt}^{\max}]$, respectively. Also, the confidence level γ in (5) is set to 0.05.

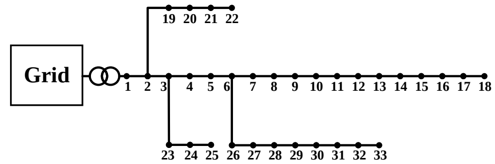

Fig. 1. The 33-bus distribution network.

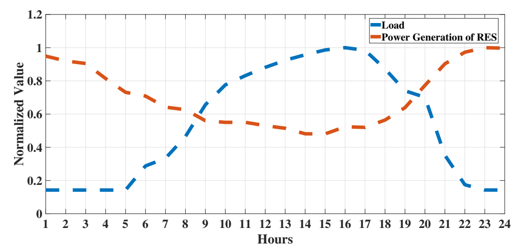

Fig. 2. The normalized patterns of loads and power generations of RESs

According to Table II, the total costs of the D-MIRP, DR-MIRP with five training in-sample scenarios, and R-MIRP models are equal to 1667 \$/day, 2155 \$/day, and 2333 \$/day, respectively. In other words, the D-MIRP model has the lowest total costs with the lowest conservatism against future realizations of the uncertain parameters while the R-MIRP model has the highest total costs with the highest conservatism against future realizations of the uncertain parameters. Additionally, more renewable generators with higher uncertainty than micro-turbine ones are constructed in the D-MIRP solution as compared to the DR-MIRP and R-MIRP ones while more micro-turbine generators with lower uncertainty than renewable ones are constructed in the R-MIRP solution as compared to the DR-MIRP and D-MIRP ones. The main reason is that the D-MIRP model excludes deviations of uncertain parameters from their nominal estimates and assumes that loads and power generations of RESs are not subject to uncertainty while the R-MIRP model includes the worst-case deviations of uncertain parameters from their nominal estimates and assumes that loads and power generations of RESs are subject to uncertainty. One the contrary, the DR-MIRP model more appropriately captures the uncertainty spectrum by means of a data-driven ambiguity set and finds a solution avoiding both the over-conservative risk-averse nature of the R-MIRP model and the risk-neutral nature of the D-MIRP model. Accordingly, the total costs of the DR-MIRP model is higher than that of

TABLE I
Characteristics of Generators

Technology	Investment Cost (\$/MW)	Operation Cost (\$/MWh)
Micro-Turbine Unit	100,000	30
Renewable Unit	120,000	5

TABLE II
OPTIMAL INVESTMENT PLANS OF DETERMINISTIC, DISTRIBUTIONALLY ROBUST, AND ROBUST MODELS

Model	Total Costs ($/day)	Built Micro-Turbines (Bus)	Built Renewable Units (Bus)	Built Lines (From Bus-To Bus)	Computation Time (s)
D-MIRP	1667	6	12, 24, 30	(1-2)	34
DR-MIRP	2155	8, 31	13, 25	(1-2), (7-8), (22-23),(30-31)	128
R-MIRP	2333	13, 25, 31	6	(1-2)	184

TABLE III
OPTIMAL INVESTMENT PLANS VERSUS THE NUMBER OF TRAINING SAMPLES

Training Sample (#)	Total Costs ($/day)	Computation Time (s)
5	2155	128
10	2141	295

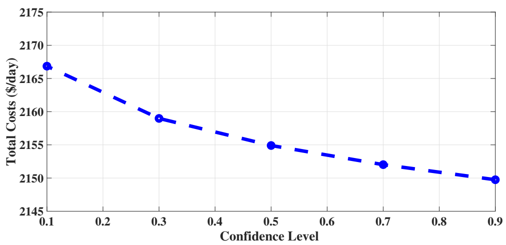

Fig. 3. Total costs versus confidence level.

the D-MIRP one and less than that of the R-MIRP model. It is worthwhile to mention that the accuracy of the DR-MIRP model can be enhanced by increasing the number of training samples. For instance, as illustrated in Table III, the total costs of the DR-MIRP model are decreased by increasing the number of training samples from 5 to 10 at the expense of a higher computation time. Also, the confidence level of the proposed DR-MIRP model can be adjusted by means of γ in (5). The variations of the total costs versus the confidence level of the proposed DR-MIRP model are demonstrated in Fig. 3. According to Fig. 3, the total costs are monotonically decreased by increasing the value of the confidence level.

V. CONCLUSIONS

In this paper, a data-driven distributionally robust model is introduced for investment/reinforcement planning in sustainable MGs under the uncertainty of loads and power generations of RESs. The proposed approach incorporates the true probability distribution of the uncertain parameters with a specific confidence level by means of a data-driven Wasserstein metric-based ambiguity set. Also, the empirical distributions of uncertain parameters as training in-sample scenarios represent the center of the Wasserstein ball. Since the proposed tri-level DR-MIRP model is intractable and it cannot be solved by available optimization packages, the equivalence between the worst-case expectation and the optimal value of the generalized moment problem as well as the strong linear programming duality are used in this paper to obtain a tractable MILP counterpart. The DR-MIRP model is compared with deterministic and robust models. Simulation results demonstrate that the proposed model is capable of capturing the uncertainty spectrum more accurate than the deterministic and robust models and avoids both the over-conservative risk-averse nature of robust model and the risk-neutral nature of the deterministic model. Also, the proposed model can control the conservatism level of the optimal solution by means of the value of the confidence level. In the future, other ambiguity sets with tractable robust counterparts can be considered.

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