A Data-Driven Optimisation Model for Designing Islanded Microgrids

Agnes M Nakiganda
Dept. of Electronic & Elec. Eng.
University of Leeds, UK
el14amn@leeds.ac.uk

Shahab Dehghan
Det. of Elec. & Electronic Eng.
Imperial College London, UK
s.dehghan@imperial.ac.uk

Petros Aristidou
Dept. of Elec. & Comp. Eng. & Informatics
Cyprus University of Technology, Cyprus
petros.aristidou@cut.ac.cy

***Abstract*—In practice, electrification of remote and islanded communities with no connection to the main grid is entangled with many techno-economic issues. These technical and more importantly economical challenges often justify the use of Micro-Grids (MGs) as self-sufficient electrical networks with a group of controllable/non-controllable consumers and producers in remote and islanded areas. However, the optimal design of sustainable MGs, even in small communities, is a complex optimisation problem due to the uncertain nature of load consumption and renewable production as well as the non-convex characteristics of network constraints. In this paper, we propose a model to design sustainable MGs using the notion of Distributionally Robust Optimisation (DRO) to handle the uncertainties arising from forecast data wherein the non-convex AC power flow equations are reformulated into convex constraints. Furthermore, a three-step approach is introduced to recast the tri-level DRO-based model into a tractable single-stage Mixed-Integer Linear Programming (MILP) problem. The proposed approach is tested on a modified European CIGRE 18-bus test network and its performance is compared with the stochastic optimisation approach.**

***Index Terms*—Distributionally Robust Optimisation, Investment Planning, Micro-Grids, Stochastic Optimisation.**

I. Introduction

A. Motivation and Background

Micro-Grids (MGs) have enabled off-grid communities to economically access electricity without the requirement for potentially high-cost long-distance energy infrastructure. Such systems have globally enhanced the electrification efforts and resilience of energy supply. Their sustainability is normally ensured by the utilisation of various Renewable Energy Sources (RESs). However, the intermittent power production of RESs adds to the level of uncertainty in the network. To ensure the reliability of the islanded MGs, system designs that remain robust to the possible adverse impacts of uncertainty are crucial. Additionally, the system security during system operation should be upheld concerning the technical limits on under/over voltage and maximum line flows. The cost-effective design of islanded MGs involves the solution of optimisation models for investment or reinforcement planning. Therefore, the handling of different uncertainties is key to the secure and resilient operation of MGs.

B. Related Research Works

Available research works on non-deterministic investment planning that account for the uncertainty of load demand and renewable power generation in active distribution networks and MGs include: Stochastic Optimisation (SO) and Robust Optimisation (RO). SO-based models obtain a solution that is optimal on average for all scenarios capturing the uncertainty spectrum [1–4]. The quality of the optimal solution in SO-based models is largely dependant on the number of available scenarios or historical data. On the contrary, RO-based models obtain a solution that is optimal for the worst scenario of a bounded uncertainty set capturing all realisations of uncertain parameters [5–7]. The uncertainty set is constructed typically assuming no distributional knowledge about the underlying uncertainty. RO usually requires less computational effort compared to SO, but provides highly conservative solutions that may result in significant over-investment.

Another non-deterministic approach bridging between RO and SO is based on Distributionally Robust Optimisation (DRO), where the optimal solution is obtained as the worst-case expected cost over a family of possible probability distribution functions (PDF) characterising the uncertain parameters in a bounded ambiguity set [8, 9]. The parameters of the ambiguity set are specified based on available distribution information, including empirical mean, variances, co-variances, distance from a known distribution [10]. Hence, the solution provided is robust against inaccuracies in the probability data. It therefore provides an intermediate and more practical solution that is less dependant on available data and less conservative. The recourse decisions in a DRO problem should adapt to all uncertain outcomes in the ambiguity set, thus making the problem generally NP hard. The nature of the ambiguity set is key in facilitating the tractable reformulations that can be solved by available numerical solvers. Different solution techniques are presented in the literature to recast DRO problems into tractable counterparts. Examples include reformulation-based approaches [11, 12] with affine policies as well as decomposition-based approaches with cutting planes [10, 13, 14].

In this paper, a DRO-based model for optimal investment planning of islanded MGs is proposed. We employ a moment-based ambiguity set due to its computational tractability as compared to other techniques [15]. Also, a data-driven approach is utilised to construct the ambiguity set where the empirical mean is inferred from historical data of energy consumption and renewable production profiles.

C. Contributions

The main contributions of this paper are three-fold:

1) We propose a formulation of a DRO-based investment planning model for islanded MGs in remote areas aimed at immunising the optimal investment plan against uncertainties in forecasted loads and renewable generations. In the proposed approach, temporal variations of loads and renewable generations during the entire planning horizon are modelled by a sufficient number of representative

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days where these representative days are extracted by the agglomerative hierarchical clustering [16]. Furthermore, a data-driven ambiguity set is presented in this paper to characterise the unknown PDFs pertaining to representative loads and renewable generations.

2) We employ the duality theory and multi-period linear decision rules (LDRs), respecting the non-anticipativity nature of the short-term operational decisions, to recast the proposed DRO-based model into a tractable mixed-integer linear programming problem (MILP).
3) We benchmark the algorithm performance against a SO-based model using the CIGRE 18-bus test network. Indices concerning computational efficiency, investment costs, and expected operational costs, are presented.

The rest of the paper is organised as follows. Section II introduces the mathematical formulation for the proposed DRO-based planning model and the definition of the ambiguity set. Section III presents the three-step approach proposed to obtain a tractable robust reformulation of model. The numerical results assessing the performance of the proposed algorithm are discussed in Section IV, while conclusions are drawn in Section V.

II. Distributionally Robust Planning Model

A. Modeling Preliminaries

Bold letters are used to indicate vectors while entries of vectors are denoted by regular letters. The transpose of a matrix is denoted by "′". This work considers a radial balanced network represented by a connected graph $\mathcal{G}(\mathcal{N},\mathcal{E})$, with $\mathcal{N}:=\{0,1,\dots,N\}$ denoting the set of network nodes including the substation node 0, and $\mathcal{E}\subseteq\mathcal{N}\times\mathcal{N}$ designating the set of network branches. The distribution network hosts a number of producers/consumers, where $\mathcal{S}\subseteq\mathcal{N}$ indicates the subset of nodes with diesel generators, $\mathcal{R}\subseteq\mathcal{N}$ the subset of nodes with RESs, $\mathcal{B}\subseteq\mathcal{N}$ the subset of nodes with battery, $\mathcal{D}\subseteq\mathcal{N}$ the subset of nodes with loads. The set of nodes with generators is thus obtained by the following set union $\mathcal{M}:=\mathcal{S}\cup\mathcal{R}\cup\mathcal{B}$. The set of respective units at node $i\in\mathcal{N}$ are given by $\mathcal{S}^i\subseteq\mathcal{S}$, $\mathcal{R}^i\subseteq\mathcal{R}$, $\mathcal{B}^i\subseteq\mathcal{B}$, and $\mathcal{D}^i\subseteq\mathcal{D}$. Cardinallity of the previously defined sets is denoted by: $n_d:=|\mathcal{D}|$, $n_s:=|\mathcal{S}|$, $n_b:=|\mathcal{B}|$, $n_l:=|\mathcal{L}|$, and $n_r:=|\mathcal{R}|$, respectively. Indices s, r, b and d are associated with diesel generators, RESs, battery units and load.

For each generator $n\in\mathcal{M}$, variables p_{nto} and q_{nto} represent active and reactive power injections; superscript "$^{\mathrm{N}}$" denotes the non-adjustable decisions terms based on forecasted parameters while superscript "$^{\mathrm{A}}$" relates to the adjustable decisions due to realisation of the forecast errors. Each node $i\in\mathcal{N}$ is connected to an upstream/downstream node $i^{\mathrm{up}}/i^{\mathrm{dn}}$ by a branch with resistance $\mathrm{r}_{ii^{\mathrm{dn}}}$ and reactance $\mathrm{x}_{ii^{\mathrm{dn}}}$, while $\mathcal{N}^{dn}$ is a set of nodes connected downstream to node i. $P_{ii^{\mathrm{dn}}}$/$Q_{ii^{\mathrm{dn}}}$ denotes the active/reactive power flow in branch $ii^{\mathrm{dn}}\in\mathcal{E}$ while S is the apparent power flow. The upper/lower limit for quantity $\bullet$ is indicated by $\overline{\bullet}/\underline{\bullet}$.

In this paper, the uncertain active renewable power generations ($r\in\mathcal{R}$) or loads ($d\in\mathcal{D}$) at timestep $t\in\mathcal{T}$ and operating condition $o\in\mathcal{O}$ is expressed as:

$$\tilde{u}_{\{r/d\}to}=u^{\mathrm{N}}_{\{r/d\}to}+\Delta\tilde{u}_{\{r/d\}to} \tag{1}$$

where $u^{\mathrm{N}}_{\{r/d\}to}$ denotes the expected/forecasted value of the power while $\Delta\tilde{u}_{\{r/d\}to}=\hat{u}_{\{r/d\}to}-\check{u}_{\{r/d\}to}$ is the forecast error where $\hat{u}_{\{r/d\}to}/\check{u}_{\{r/d\}to}$ denotes the upward/downward deviation from the forecast value. As uncertainties of both electricity consumption and renewable production are considered, $\tilde{u}_{\{r/d\}to}$ is defined as:

$$\tilde{u}_{to}=\left\{\begin{array}{ll}\tilde{p}_{dto}=p^{\mathrm{N}}_{dto}+\hat{p}_{dto}-\check{p}_{dto}, & \forall d\\ \tilde{p}_{rto}=p^{\mathrm{N}}_{rto}+\hat{p}_{rto}-\check{p}_{rto}, & \forall r\end{array}\right\},\quad\forall t,o \tag{2}$$

where $\tilde{p}_{dto}$ relates to the uncertain loads and $\tilde{p}_{rto}$ relates to the uncertain renewable generations. In this work, a constant load power factor is considered where $\cos\theta_{dto}=\frac{p^{\mathrm{N}}_{dto}}{\sqrt{(p^{\mathrm{N}}_{dto})^2+(q^{\mathrm{N}}_{dto})^2}}$. Therefore, the uncertain reactive loads are defined as: $\tilde{q}_{dto}=tan\theta_{dto}\cdot\tilde{p}_{dto}$. Similarly, a constant reactive power control for RESs is adopted, i.e., $-\tan\overline{\phi}_r\cdot\tilde{p}_{rto}\leq q_{rto}\leq\tan\overline{\phi}_r\cdot\tilde{p}_{rto}$, where parameter $\cos\overline{\phi}_r$ is the minimum power factor set by the system operator. The uncertain reactive power is therefore defined as: $\tilde{q}_{rto}=\tan\overline{\phi}_r\cdot\tilde{p}_{rto}$. Note that uncertain reactive power injection/absorption is a function of the uncertain active power and not required to be defined explicitly.

B. Ambiguity Set Model

The compact form of the tri-level DRO model is presented as:

$$\min_{\chi^{\mathrm{inv}},\chi^{\mathrm{opr}}}\left\{\Theta^{\mathrm{inv}}(\chi^{\mathrm{inv}})+\max_{\mathbb{P}\in\mathcal{U}}\mathbb{E}_{\mathbb{P}}\left(\Theta^{\mathrm{opr}}(\chi^{\mathrm{opr}},\tilde{\mathbf{u}})\right)\right\} \tag{3}$$

where $\Theta^{\mathrm{inv}}/\Theta^{\mathrm{opr}}$ are the the investment/operational costs and $\chi^{\mathrm{inv}}/\chi^{\mathrm{opr}}$ the vectors of investment/operational variables while $\mathbb{E}_{\mathbb{P}}$ calculates the expected value of the operational costs. Also, $\tilde{\mathbf{u}}$ defines the vector of the uncertain variables while the ambiguity set $\mathcal{U}$ characterises the distribution of the uncertain parameters for the entire planning horizon and is obtained as a Cartesian product of the set at each time step for all operating scenarios:

$$\mathcal{U}=\prod_{t\in T,o\in\mathcal{O}}\mathcal{U}_{to} \tag{4}$$

where

$$\mathcal{U}_{to}=\left\{\mathbb{P}_{to}:\begin{array}{ll}\mathbb{E}_{\mathbb{P}_{to}}(\tilde{p}_{dto})=p^{\mathrm{N}}_{dto}, & \forall d\\ \mathbb{E}_{\mathbb{P}_{to}}(\tilde{p}_{rto})=p^{\mathrm{N}}_{rto}, & \forall r\\ \mathbb{P}_t\left\{\begin{array}{l}\tilde{p}_{dto}\in\mathcal{V}_{to}\\ \tilde{p}_{rto}\in\mathcal{V}_{to}\end{array}\right\}=1, & \end{array}\right\} \tag{5}$$

In (5) the first and second lines indicate that the mean of the uncertain parameters is defined by their respective forecast values while the third line guarantees that all realisations of uncertainties are within the uncertainty set $\mathcal{V}_{to}$. We adopt the polyhedral uncertainty set proposed in [17] where a budget of uncertainty Γ is used to control the conservatism. $\mathcal{V}_{to}$ is expressed by constraints:

$$\mathcal{V}_{to}=\left\{\begin{array}{ll}\tilde{p}_{dto}=p^{\mathrm{N}}_{dto}+\hat{p}_{dto}-\check{p}_{dto}, & \forall d\\ \tilde{p}_{rto}=p^{\mathrm{N}}_{rto}+\hat{p}_{rto}-\check{p}_{rto}, & \forall r\\ 0\leq\hat{p}_{dto}\leq\overline{\hat{p}}_{dto},\ 0\leq\check{p}_{dto}\leq\overline{\check{p}}_{dto}, & \forall d\\ 0\leq\hat{p}_{rto}\leq\overline{\hat{p}}_{rto},\ 0\leq\check{p}_{rto}\leq\overline{\check{p}}_{rto}, & \forall r\\ 0\leq\left(\sum_{d\in\mathcal{D}}\left(\frac{\hat{p}_{dto}}{\overline{\hat{p}}_{dto}}+\frac{\check{p}_{dto}}{\overline{\check{p}}_{dto}}\right)\right. & \\ \quad\left.+\sum_{r\in\mathcal{R}}\left(\frac{\hat{p}_{rto}}{\overline{\hat{p}}_{rto}}+\frac{\check{p}_{rto}}{\overline{\check{p}}_{rto}}\right)\right)\leq\Gamma_{to} & \end{array}\right\} \tag{6}$$

C. Investment Planning Model

We expand the formulation of the proposed DRO-based planning model in (3) as follows:

1) Objective: The term $\Theta^{\text{inv}}(\chi^{\text{inv}})$ in (3) is given by:

$$\Theta^{\text{inv}} = \sum_{b\in\mathcal{B}} C_b \cdot z_b + \sum_{s\in\mathcal{S}} C_s \cdot z_s + \sum_{r\in\mathcal{R}} C_r \cdot z_r \tag{7a}$$

where $C_{b/s/r}$ is the investment cost of a particular unit and $z_{b/s/r}$ is the binary variable indicating the investment status of a unit. Also, the term $\Theta^{\text{opr}}(\chi^{\text{opr}})$ in (3) is defined as:

$$\Theta^{\text{opr}} = \sum_{o\in\mathcal{O}} \sum_{t\in\mathcal{T}} \Bigg(\sum_{s\in\mathcal{S}} C_s^{\text{op}} \cdot p_{sto} + \sum_{r\in\mathcal{R}} C_r^{\text{op}} \cdot p_{rto} + \sum_{d\in\mathcal{D}} C_d^{\text{sh}} \cdot \tilde{p}_{dto} \cdot (1 - z_{dto}) + \sum_{i\in\mathcal{N}} \epsilon \cdot q_{ito}^{\text{aux}} \Bigg) \tag{7b}$$

here $C_{s/r}^{\text{op}}$ is the marginal operational cost of each unit while C_d^{sh} is the penalty cost of load shedding. Variable z_{dto} is used to indicate the connection status of a load. To ensure the nodal reactive power balance, a small cost ϵ has been applied to the magnitude of reactive power generation denoted by the auxiliary variable q_{ito}^{aux}. In the following, a definition of the constraints applied to the model is presented.

2) Power Flow Constraints: A linearized version of the 'DistFlow' model [18] is used to formulate the power flow equations in (7c)-(7e), where v_{ito} denotes the square magnitude of voltage at each node $i \in \mathcal{N}$, time period $t \in \mathcal{T}$, and operating condition $o \in \mathcal{O}$:

$$\sum_{s\in\mathcal{S}^i} p_{sto} + \sum_{r\in\mathcal{R}^i} p_{rto} + \sum_{b\in\mathcal{B}^i} (p_{bto}^{\text{dch}} - p_{bto}^{\text{ch}}) + P_{i^{\text{up}}ito} - \sum_{i^{\text{dn}}\in\mathcal{N}^{dn}} P_{ii^{\text{dn}}to} \geq \sum_{d\in\mathcal{D}^i} \tilde{p}_{dto} \cdot z_{dto}, \quad \forall i,t,o \tag{7c}$$

$$\sum_{s\in\mathcal{S}^i} q_{sto} + \sum_{r\in\mathcal{R}^i} q_{rto} + Q_{i^{\text{up}}ito} - \sum_{i^{\text{dn}}\in\mathcal{N}^{dn}} Q_{ii^{\text{dn}}to} \geq \sum_{d\in\mathcal{D}^i} \tilde{q}_{dto} \cdot z_{dto}, \quad \forall i,t,o \tag{7d}$$

$$v_{i^{\text{up}}to} = v_{ito} + 2\left(r_{i^{\text{up}}i} \cdot P_{i^{\text{up}}ito} + x_{i^{\text{up}}i} \cdot Q_{i^{\text{up}}ito}\right), \quad \forall i,t,o \tag{7e}$$

$$-q_{ito}^{\text{aux}} \leq \sum_{s\in\mathcal{S}^i} q_{sto} + \sum_{r\in\mathcal{R}^i} q_{rto} \leq q_{ito}^{\text{aux}}, \quad \forall i,t,o \tag{7f}$$

$$q_{ito}^{\text{aux}} \geq 0, \quad \forall i,t,o \tag{7g}$$

where superscript "$^{\text{ch}}$/$^{\text{dch}}$" indicates the charge/discharge power of the battery units. Different generators in the network have the capability to inject/absorb reactive power. It is required that the nodal reactive power balance is respected, given the mode of operation, i.e., injection/absorption. This is ensured when equality exists between the left-hand-side and right-hand-side of (7d). This requirement is met using the non-negative auxiliary variable q_{ito}^{aux} in (7f) to which a small cost is applied in the objective function such that equality in (7d) is maintained.

3) Dispatchable Generation Constraints: Diesel units are fully dispatchable while renewable units are assumed to be dispatchable-down within their capacity limits.

$$0 \leq p_{sto} \leq \overline{p}_s \cdot z_s, \quad -\overline{q}_s \cdot z_s \leq q_{sto} \leq \overline{q}_s \cdot z_s, \ \forall s,t,o \tag{7h}$$

$$-\text{rp}_s^{\text{dn}} \leq p_{sto} - p_{s(t-1)o} \leq \text{rp}_s^{\text{up}}, \quad \forall s,t,o \tag{7i}$$

$$0 \leq p_{rto} \leq \tilde{p}_{rto} \cdot z_r, \quad \forall r,t,o \tag{7j}$$

$$-\tan\overline{\phi} \cdot \tilde{p}_{rto} \cdot z_r \leq q_{rto} \leq \tan\overline{\phi} \cdot \tilde{p}_{rto} \cdot z_r, \quad \forall r,t,o \tag{7k}$$

Binary variable z_s/z_r indicates the investment status of the diesel/renewable unit limited by its maximum active /reactive power capacity denoted by $\overline{p}/\overline{q}$. The maximum capacity of each renewable unit is equal to the available usable power of the unit at a given time. The maximum ramp up/down limits $\text{rp}_s^{\text{up}}/\text{rp}_s^{\text{dn}}$ are defined in (7i).

4) Constraints of Battery Units: Constraint (7l) limits the charging/discharging power of battery units within their charge/discharge capacities while (7m) prevents simultaneous charging z_{bto}^{ch} and discharging z_{bto}^{dch} of the battery given its investment status z_b. The battery state-of-charge (SOC) at each hour is limited by the maximum/minimum energy limit $\overline{e}_b/\underline{e}_b$ in (7n), while the initial (e_{bo}^{ini}) and final SOC are set by constraint (7o), given charging/discharging efficiency $\xi_b^{\text{ch}}/\xi_b^{\text{dch}}$.

$$0 \leq p_{bto}^{\text{dch}} \leq \overline{p}_b^{\text{dch}} \cdot z_{bto}^{\text{dch}}, \ 0 \leq p_{bto}^{\text{ch}} \leq \overline{p}_b^{\text{ch}} \cdot z_{bto}^{\text{ch}}, \ \forall b,t,o \tag{7l}$$

$$z_{bto}^{\text{dch}} + z_{bto}^{\text{ch}} \leq z_b, \quad \forall b,t,o \tag{7m}$$

$$\underline{e}_b \cdot z_b \leq e_{bo}^{\text{ini}} + \sum_{\tau=1}^{t} \left(\xi_b^{\text{ch}} \cdot p_{b\tau o}^{\text{ch}} - \frac{1}{\xi_b^{\text{dch}}} \cdot p_{b\tau o}^{\text{dch}} \right) \leq \overline{e}_b \cdot z_b, \quad \forall b,t,o \tag{7n}$$

$$\sum_{t\in\mathcal{T}} \left(\xi_b^{\text{ch}} \cdot p_{bto}^{\text{ch}} - \frac{1}{\xi_b^{\text{dch}}} \cdot p_{bto}^{\text{dch}} \right) = 0, \quad \forall b,o \tag{7o}$$

5) Thermal Loading and Voltage Constraints: Quadratic constraint (7p) denotes the secure line loading limits. They are linearized using a piece-wise linear approximation approach defined in [19], while (7q) defines the limits on nodal voltages.

$$(P_{i^{\text{up}}ito})^2 + (Q_{i^{\text{up}}ito})^2 \leq (S_{i^{\text{up}}i})^2, \quad \forall i,t,o \tag{7p}$$

$$\underline{v} \leq v_{ito} \leq \overline{v}, \quad v_{to|i=0} = 1, \quad \forall i,t,o \tag{7q}$$

D. Compact Matrix Formulation

For a clear presentation, the overall formulation can be presented as a compact matrix expressed below:

$$\min_{\chi^{\text{inv}},\chi^{\text{opr}}} \left\{ \Theta^{\text{inv}}(\chi^{\text{inv}}) + \max_{\mathbb{P}\in\mathcal{U}} \mathbb{E}_{\mathbb{P}}\left(\Theta^{\text{opr}}(\chi^{\text{opr}}, \tilde{\boldsymbol{u}})\right) \right\} \tag{8a}$$

$$\text{s.t. } \boldsymbol{A}\chi^{\text{inv}} + \boldsymbol{B}\boldsymbol{h}(\chi^{\text{opr}}, \tilde{\boldsymbol{u}}) \geq \boldsymbol{q} + \boldsymbol{Q}\tilde{\boldsymbol{u}}, \quad \forall \tilde{\boldsymbol{u}} \in \mathcal{V} \tag{8b}$$

Constraints (7c)-(7q) are generalised into (8b) where function $h(\chi^{\text{opr}}, \tilde{\boldsymbol{u}})$ is associated with the effect of the uncertain parameters on the decision variables during system operation, while $\boldsymbol{A}$, $\boldsymbol{B}$, $\boldsymbol{q}$ and $\boldsymbol{Q}$ are constant matrices. Set $\mathcal{V}$ is the uncertainty set defined in (6).

E. Transformation of the Worst-Case Expectation

Based on the definition of the ambiguity set $\mathcal{U}$ in (5), the worst-case expectation in objective of the operational problem in (8a) can be explicitly represented as:

$$\max_{\mathbb{P}\in\mathcal{U}} \mathbb{E}_{\mathbb{P}}\left(\Theta^{\text{opr}}(\chi^{\text{opr}}, \tilde{\boldsymbol{u}})\right) = \max \int_{\mathcal{V}} \Theta^{\text{opr}}(\chi^{\text{opr}}, \tilde{\boldsymbol{u}}) \, \mathrm{d}P(\tilde{\boldsymbol{u}}) \tag{9a}$$

$$\text{s.t} \int_{\mathcal{V}} \tilde{\boldsymbol{u}} \, \mathrm{d}P(\tilde{\boldsymbol{u}}) = \boldsymbol{u}^{\text{N}} \qquad (\text{dual } \boldsymbol{\eta}) \tag{9b}$$

$$\int_{\mathcal{V}} \mathrm{d}P(\tilde{\boldsymbol{u}}) = 1 \qquad (\text{dual } \boldsymbol{\beta}) \tag{9c}$$

$$\mathrm{d}P(\tilde{\boldsymbol{u}}) \geq 0, \quad \forall \tilde{\boldsymbol{u}} \in \mathcal{V} \tag{9d}$$

where the decision variable $P(\tilde{\boldsymbol{u}})$ is the probability distribution function; while $\boldsymbol{\eta}$ and $\boldsymbol{\beta}$ are vectors of dual variables associated with constraints (9b) and (9c), respectively. Using the duality theory [20], (9) can be recast into (10) as indicated below:

$$\max_{\mathbb{P}\in\mathcal{U}} \mathbb{E}_{\mathbb{P}}\left(\Theta^{\text{opr}}\right) = \min \left(\boldsymbol{\beta} + \boldsymbol{\eta}'\boldsymbol{u}^{\text{N}} \right) \tag{10a}$$

$$\text{s.t. } \boldsymbol{\beta}+\boldsymbol{\eta}'\tilde{\boldsymbol{u}} \geq \Theta^{\text{opr}}(\chi^{\text{opr}},\tilde{\boldsymbol{u}}),\ \forall\tilde{\boldsymbol{u}}\in\mathcal{V} \quad (10b)$$

The model can now be represented as:

$$\min \left(\Theta^{\text{inv}}+\boldsymbol{\beta}+\boldsymbol{\eta}'\boldsymbol{u}^{\text{N}}\right) \quad (11a)$$

$$\text{s.t. } \boldsymbol{\beta}+\boldsymbol{\eta}'\tilde{\boldsymbol{u}} \geq \Theta^{\text{opr}}(\chi^{\text{opr}},\tilde{\boldsymbol{u}}) \qquad \forall\tilde{\boldsymbol{u}}\in\mathcal{V} \quad (11b)$$

$$\boldsymbol{A}\chi^{\text{inv}}+\boldsymbol{B}\boldsymbol{h}(\chi^{\text{opr}},\tilde{\boldsymbol{u}}) \geq \boldsymbol{q}+\boldsymbol{Q}\tilde{\boldsymbol{u}} \qquad \forall\tilde{\boldsymbol{u}}\in\mathcal{V} \quad (11c)$$

Note that the model in (11b) contains a bilinear term $\boldsymbol{\eta}'\tilde{\boldsymbol{u}}$ resulting in a non-convex formulation that is NP hard. Additionally, (11) is intractable due to its infinite-dimensional nature, i.e., it should be feasible for any realisation of the uncertain parameters whose coverage is defined by the ambiguity set in (5). In this work we utilise decision rules and duality theory to recast the problem to its robust counterpart.

III. Solution Approach

In this section, we present a three-step procedure to derive a tractable robust counterpart for the proposed problem that can be easily solved by off-the-shelf solvers.

A. Defining the Decision Rules

LDRs restrict the recourse decisions to affine functions of the uncertain parameters [21]. It is noteworthy to mention that by its nature, the decision-making process involves multiple stages, i.e., the decisions made at each time step are dependant on the decisions made at the previous time steps. Disregarding this dependency in the decision rule at each time step could violate the nonanticipativity constraints present in the model. In this work, these constraints relate to the inter-temporal constraints on the ramping limits of the generators (7i) and battery state-of-charge at the end of planning horizon (7n)-(7o).

In the first step, we formulate a nonanticipative LDR for the independent variables, i.e., hourly active/reactive power injection/absorption of different types of units. The voltage levels, currents, and power flows are dependant on the power injection/absorption, hence, do not require explicit LDRs. The active/reactive power policies for each unit $n\in\mathcal{S}\cup\mathcal{R}\cup\mathcal{B}$ are thus defined as:

$$p_{nto}=p^{\text{N}}_{nto}+\sum_{k=1}^{t}\left(\sum_{d\in\mathcal{D}}\left(\hat{p}^{\text{AD}}_{ndkto}\cdot\hat{p}_{dko}-\check{p}^{\text{AD}}_{ndkto}\cdot\check{p}_{dko}\right)+\sum_{r\in\mathcal{R}}\left(-\hat{p}^{\text{AR}}_{nrkto}\cdot\hat{p}_{rko}+\check{p}^{\text{AR}}_{nrkto}\cdot\check{p}_{rko}\right)\right) \quad (12a)$$

$$q_{nto}=q^{\text{N}}_{nto}+\sum_{k=1}^{t}\left(\sum_{d\in\mathcal{D}}\left(\hat{q}^{\text{AD}}_{ndkto}\cdot\hat{p}_{dko}-\check{q}^{\text{AD}}_{ndkto}\cdot\check{p}_{dko}\right)\cdot\tan\theta_{dko}+\sum_{r\in\mathcal{R}}\left(-\hat{q}^{\text{AD}}_{nrkto}\cdot\hat{p}_{rko}+\check{q}^{\text{AD}}_{nrkto}\cdot\check{p}_{rko}\right)\cdot\tan\overline{\phi}\right) \quad (12b)$$

Superscripts "$^{\text{D}}$" and "$^{\text{R}}$" relate to variables associated with demand-related and renewable-related uncertainty, respectively. The rule definitions in (12) expressing the effect of the uncertain parameters can be compactly represented as:

$$h(\chi^{\text{opr}},\tilde{\boldsymbol{u}})=\chi^{\text{opr,N}}+\chi^{\text{opr,A}}\Delta\tilde{\boldsymbol{u}} \quad (13)$$

where $\chi^{\text{opr,N}}/\chi^{\text{opr,A}}$ denotes the vector/matrix of non-adjustable/adjustable variables. Also, $\Delta\tilde{\boldsymbol{u}}=(\Delta\tilde{\boldsymbol{u}}^{1\prime},...,\tilde{\boldsymbol{u}}^{t\prime},...,\Delta\tilde{\boldsymbol{u}}^{T\prime})$ represents the vector of uncertain parameters for all hours of the planning horizon where the vector $\tilde{\boldsymbol{u}}^{t}$ includes all uncertain parameters from hour 1 to t.

The adjustable variables perform as proxies in finding the worst expected costs. In practice a non-zero value for an adjustable variable represents a variation from the forecast value, and consequently, an additional cost in the objective function.

B. Problem Reformulation using LDRs

In the second step, the problem is reformulated by the LDRs. The rule defined in (13) is then applied to the model as follows:

$$\min \left(\Theta^{\text{inv}}+\boldsymbol{\beta}+\boldsymbol{\eta}'\boldsymbol{u}^{\text{N}}\right) \quad (14a)$$

$$\text{s.t. } \boldsymbol{\beta}\geq\boldsymbol{C}\chi^{\text{opr,N}}+\boldsymbol{C}\chi^{\text{opr,A}}\Delta\tilde{\boldsymbol{u}}-\boldsymbol{\eta}'\tilde{\boldsymbol{u}}, \quad \forall\tilde{\boldsymbol{u}}\in\mathcal{V} \quad (14b)$$

$$\boldsymbol{A}\chi^{\text{inv}}+\boldsymbol{B}\chi^{\text{opr,N}}-\boldsymbol{q} \geq \boldsymbol{Q}\tilde{\boldsymbol{u}}-\boldsymbol{B}\chi^{\text{opr,A}}\Delta\tilde{\boldsymbol{u}}, \qquad \forall\tilde{\boldsymbol{u}}\in\mathcal{V} \quad (14c)$$

where $\Theta^{\text{opr}}(\chi^{\text{opr}},\tilde{\boldsymbol{u}})$ is reformulated as $\Theta^{\text{opr}}(\chi^{\text{opr}},\tilde{\boldsymbol{u}})=\boldsymbol{C}\chi^{\text{opr,N}}+\boldsymbol{C}\chi^{\text{opr,A}}\Delta\tilde{\boldsymbol{u}}$. However, the optimisation problem (14) is still intractable due to the universal quantifier over the vector uncertain parameters (i.e., $\forall\tilde{\boldsymbol{u}}\in\mathcal{V}$). To obtain a robust solution against any realisation of uncertain parameters, a worst-case reformulation is introduced in this paper using the protection functions $\Phi^1(\tilde{\boldsymbol{u}})$ and $\Phi^2(\tilde{\boldsymbol{u}})$ as given below:

$$\min \left(\Theta^{\text{inv}}+\boldsymbol{\beta}+\boldsymbol{\eta}'\boldsymbol{u}^{\text{N}}\right) \quad (15a)$$

$$\text{s.t. } \boldsymbol{\beta}-\boldsymbol{C}\chi^{\text{opr,N}}\geq\underbrace{\max_{\tilde{\boldsymbol{u}}\in\mathcal{V}}\left(\boldsymbol{C}\chi^{\text{opr,A}}\Delta\tilde{\boldsymbol{u}}-\boldsymbol{\eta}'\tilde{\boldsymbol{u}}\right)}_{\Phi^1(\tilde{\boldsymbol{u}})} \quad (15b)$$

$$\boldsymbol{A}\chi^{\text{inv}}+\boldsymbol{B}\chi^{\text{opr,N}}-\boldsymbol{q} \geq \underbrace{\max_{\tilde{\boldsymbol{u}}\in\mathcal{V}}\left(\boldsymbol{Q}\tilde{\boldsymbol{u}}-\boldsymbol{B}\chi^{\text{opr,A}}\Delta\tilde{\boldsymbol{u}}\right)}_{\Phi^2(\tilde{\boldsymbol{u}})} \quad (15c)$$

The protection functions $\Phi^1(\tilde{\boldsymbol{u}})$ and $\Phi^2(\tilde{\boldsymbol{u}})$ for constraints (15b) and (15c) depend on the polyhedral uncertainty set $\mathcal{V}$ defined in (6), they can be rewritten as:

$$\Phi^1(\tilde{\boldsymbol{u}})=\max_{\tilde{\boldsymbol{u}}\in\mathcal{V}}\left(\left(\boldsymbol{C}\chi^{\text{opr,A}}\hat{\boldsymbol{u}}-\boldsymbol{\eta}'\hat{\boldsymbol{u}}\right)-\left(\boldsymbol{C}\chi^{\text{opr,A}}\check{\boldsymbol{u}}-\boldsymbol{\eta}'\check{\boldsymbol{u}}\right)-\boldsymbol{\eta}'\boldsymbol{u}^{\text{N}}\right) \quad (16a)$$

$$\Phi^2(\tilde{\boldsymbol{u}})=\max_{\tilde{\boldsymbol{u}}\in\mathcal{V}}\left(\left(\boldsymbol{Q}\hat{\boldsymbol{u}}-\boldsymbol{B}\chi^{\text{opr,A}}\hat{\boldsymbol{u}}\right)-\left(\boldsymbol{Q}\check{\boldsymbol{u}}-\boldsymbol{B}\chi^{\text{opr,A}}\check{\boldsymbol{u}}\right)+\boldsymbol{Q}\boldsymbol{u}^{\text{N}}\right) \quad (16b)$$

$$\text{s.t. } 0\leq\hat{\boldsymbol{u}}\leq\overline{\hat{\boldsymbol{u}}} \qquad (\text{dual } \boldsymbol{\lambda}) \quad (16c)$$

$$0\leq\check{\boldsymbol{u}}\leq\overline{\check{\boldsymbol{u}}} \qquad (\text{dual } \boldsymbol{\pi}) \quad (16d)$$

$$\hat{\boldsymbol{u}}/\overline{\hat{\boldsymbol{u}}}+\check{\boldsymbol{u}}/\overline{\check{\boldsymbol{u}}}=\boldsymbol{\Gamma} \qquad (\text{dual } \boldsymbol{\psi}) \quad (16e)$$

where $\boldsymbol{\lambda}$, $\boldsymbol{\pi}$ and $\boldsymbol{\psi}$ are vectors of the dual variables associated with constraints (16c)-(16e).

C. Applying Duality Theory

Finally, in the third step, we use duality theory to obtain a tractable reformulation of the problem. The maximisation problem in (16a) can be recast into a minimisation problem using the duality theory as follows:

$$\Phi^1(\tilde{\boldsymbol{u}})=\min\left(\left(\overline{\hat{\boldsymbol{u}}}'\boldsymbol{\lambda}^1+\overline{\check{\boldsymbol{u}}}'\boldsymbol{\pi}^1+\boldsymbol{\Gamma}'\psi^1\right)-\boldsymbol{\eta}'\boldsymbol{u}^{\text{N}}\right) \quad (17a)$$

$$\text{s.t. } \boldsymbol{\lambda}^1+\left(1/\overline{\hat{\boldsymbol{u}}}\right)'\psi^1\geq\left((\chi^{\text{opr,A}})'\boldsymbol{C}'-\boldsymbol{\eta}\right) \quad (17b)$$

$$\boldsymbol{\pi}^1 + \left(1/\overline{\tilde{\boldsymbol{u}}}\right)' \boldsymbol{\psi}^1 \geq -\left((\chi^{\text{opr,A}})'\boldsymbol{C}' - \boldsymbol{\eta}\right) \tag{17c}$$

Similarly, the maximisation problem in (16b) can be recast into a minimisation problem as follows:

$$\Phi^2(\tilde{\boldsymbol{u}}) = \min\left(\left(\overline{\tilde{\boldsymbol{u}}}'\boldsymbol{\lambda}^2 + \overline{\tilde{\boldsymbol{u}}}'\boldsymbol{\pi}^2 + \boldsymbol{\Gamma}'\boldsymbol{\psi}^2\right) + \boldsymbol{Q}\boldsymbol{u}^{\text{N}}\right) \tag{18a}$$

$$\text{s.t. } \boldsymbol{\lambda}^2 + \left(1/\overline{\tilde{\boldsymbol{u}}}\right)' \boldsymbol{\psi}^2 \geq \left(\boldsymbol{Q} - \boldsymbol{B}\chi^{\text{opr,A}}\right)' \tag{18b}$$

$$\boldsymbol{\pi}^2 + \left(1/\overline{\tilde{\boldsymbol{u}}}\right)' \boldsymbol{\psi}^2 \geq -\left(\boldsymbol{Q} - \boldsymbol{B}\chi^{\text{opr,A}}\right)' \tag{18c}$$

Superscripts "^{1}" and "^{2}" are utilised to distinguish between the dual variables in (17) and (18), respectively. Therefore, the overall problem is reformulated as:

$$\min \left(\Theta^{\text{inv}} + \boldsymbol{\beta} + \boldsymbol{\eta}'\boldsymbol{u}^{\text{N}}\right) \tag{19a}$$

$$\text{s.t. } \boldsymbol{\beta} - \boldsymbol{C}\chi^{\text{opr,N}} \geq \left(\overline{\tilde{\boldsymbol{u}}}'\boldsymbol{\lambda}^1 + \overline{\tilde{\boldsymbol{u}}}'\boldsymbol{\pi}^1 + \boldsymbol{\Gamma}'\boldsymbol{\psi}^1\right) - \boldsymbol{\eta}'\boldsymbol{u}^{\text{N}} \tag{19b}$$

$$\boldsymbol{A}\chi^{\text{inv}} + \boldsymbol{B}\chi^{\text{opr,N}} - \boldsymbol{q} \geq \left(\overline{\tilde{\boldsymbol{u}}}'\boldsymbol{\lambda}^2 + \overline{\tilde{\boldsymbol{u}}}'\boldsymbol{\pi}^2 + \boldsymbol{\Gamma}'\boldsymbol{\psi}^2\right) + \boldsymbol{Q}\boldsymbol{u}^{\text{N}} \tag{19c}$$

$$\boldsymbol{\lambda}^1 + \left(1/\overline{\tilde{\boldsymbol{u}}}\right)' \boldsymbol{\psi}^1 \geq \left((\chi^{\text{opr,A}})'\boldsymbol{C}' - \boldsymbol{\eta}\right) \tag{19d}$$

$$\boldsymbol{\pi}^1 + \left(1/\overline{\tilde{\boldsymbol{u}}}\right)' \boldsymbol{\psi}^1 \geq -\left((\chi^{\text{opr,A}})'\boldsymbol{C}' - \boldsymbol{\eta}\right) \tag{19e}$$

$$\boldsymbol{\lambda}^2 + \left(1/\overline{\tilde{\boldsymbol{u}}}\right)' \boldsymbol{\psi}^2 \geq \left(\boldsymbol{Q} - \boldsymbol{B}\chi^{\text{opr,A}}\right)' \tag{19f}$$

$$\boldsymbol{\pi}^2 + \left(1/\overline{\tilde{\boldsymbol{u}}}\right)' \boldsymbol{\psi}^2 \geq -\left(\boldsymbol{Q} - \boldsymbol{B}\chi^{\text{opr,A}}\right)' \tag{19g}$$

$$\boldsymbol{\lambda}^1 \geq 0,\ \boldsymbol{\pi}^1 \geq 0,\ \boldsymbol{\lambda}^2 \geq 0,\ \boldsymbol{\pi}^2 \geq 0 \tag{19h}$$

The problem formulation in (19) is a single-level MILP problem that can tractably be solved by various available off-shelf solvers.

IV. Case Studies

A. Test System Setup

The data-driven DRO-based planning model described above is tested on a modified European CIGRE low-voltage network [22] sketched in Fig. 1. It is assumed that the network is operated in islanded mode with no connection to the grid. One diesel unit (SG) is already installed at node 1. The investment candidates include three Photo-Voltaic (PV) units PV_1 and PV_2 and PV_3; three Energy Storage (ES) units denoted ES_1 and ES_2 and ES_3; three SG units SG_1 and SG_2 and SG_3, with the capacity of each set at 0.55 MW. Candidate units with subscripts "$_1$", "$_2$" and "$_3$" are located at nodes 11, 17 and 18, respectively. The investment and operational costs are shown in Table. I. For the annualised costs, an interest rate of 0.053 is assumed, and the life time of ES, PV, and wind units is set at 15, 30, and 30 years, respectively. The load and renewable generation profiles have been obtained from [23] using the UK values in 2019. A 24-hour planning horizon is considered for each representative day. The computation was performed in Python using Pyomo [24] to model the optimisation problem and Gurobi [25] employed as a solver.

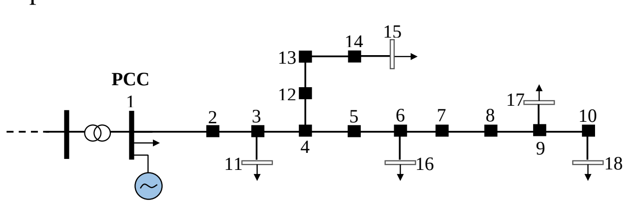

Fig. 1. Modified European CIGRE low voltage network.

TABLE I
INVESTMENT COSTS OF DIFFERENT TECHNOLOGIES

Technology	Battery (ES)	Solar (PV)	Diesel (SG)
Investment Cost [M£/MW]	0.98	0.84	0.54
Annualized Investment Cost [£/MW]	96040	56280	36180
Operation Cost [£/MWh]	-	0	150

TABLE II
VARIATION OF INVESTMENT COSTS, DECISIONS, AND OPERATING COSTS WITH THE BUDGET OF UNCERTAINTY

Budget [Γ]	Investment Cost [M£]	Operation Cost [M£]	Investment Decisions			Comp. Time [s]
			PV	ES	SG	
0	0.0310	0.1283	PV_3	-	-	108
1	0.0929	0.1607	PV_1, PV_2, PV_3	-	-	222
2	0.0929	0.3361	PV_1, PV_2, PV_3	-	-	342
3	0.0929	0.4986	PV_1, PV_2, PV_3	-	-	452
4	0.0929	0.5404	PV_1, PV_2, PV_3	-	-	571
5	0.1102	0.5715	PV_1, PV_2, PV_3	-	SG_3	683
6	0.1102	0.5848	PV_1, PV_2, PV_3	-	SG_3	804
7	0.1102	0.5848	PV_1, PV_2, PV_3	-	SG_3	919
8	0.1102	0.5848	PV_1, PV_2, PV_3	-	SG_3	1040

B. Optimal Solution Versus Budget of Uncertainty

The robustness and thus conservatism of the model can be varied by the budget of uncertainty. A higher budget of uncertainty corresponds to the widening of the uncertainty spectrum captured in the model parameters. In the study network, a maximum value of nine includes the forecast errors of both the loads (six) and renewable generations (three) available. In Table II, the effect of an increase in the budget of uncertainty to the investment decisions and operating costs is presented considering two representative days. It should be noted that the case of zero budget of uncertainty is similar to the stochastic solution of the problem. Both the total investment and operational costs are seen to increase with the former reaching a plateau at a value of four while the latter becomes constant at a value of six. At zero, a total cost of 0.1503 M£ is recorded compared to a value of 0.695 M£ at the maximum budget of uncertainty. While the maximum value of the budget of uncertainty captures all potential forecast errors within the ambiguity set, it can be rather conservative.

C. Optimal Solution Versus Number of Representative Days

By increasing the number of representative days in the ambiguity set of a DRO problem, the distributional nature of the uncertainty is better captured. In Fig. 2, the result of variation of the number of representative days is presented for both the proposed DRO model and the SO model. The budget of uncertainty for the DRO model is set to four in this case study. The total costs in the proposed DRO model are shown to reduce with an increase in the number of representative days with total costs recorded at 0.5139 M£ at four representative days compared to 0.3924 M£ at ten representative days. While the investment costs increase with more representative days, the operational costs indicate a decline. Table III presents the investment decisions taken under DRO and SO uncertainty handling. With more representative days, the available usable power from the renewable sources is better represented and thus more usable. Additionally, the variations in forecasted load profiles are better represented with the increased operational scenarios i.e., representative days. The load variations require flexibility in available generation. This

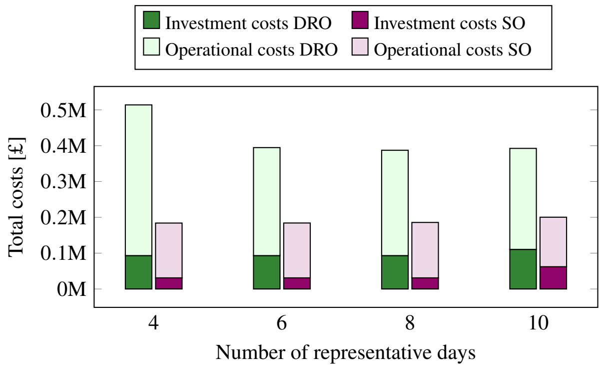

Fig. 2. Total costs under different number of representative days for DRO and SO models.

TABLE III
INVESTMENT DECISIONS UNDER DRO AND SO MODELS FOR INCREASING REPRESENTATIVE DAYS.

	DRO		SO	
Rep. Days	Decision	Comp. Time [s]	Decision	Comp. Time [s]
4	PV_1, PV_2, PV_3	109	PV_1	44
6	PV_1, PV_2, PV_3	333	PV_1	118
8	PV_1, PV_2, PV_3	682	PV_1	217
10	PV_1, PV_2, PV_3, SG_3	1175	PV_1, PV_2	476

flexibility requirement is fulfilled by the installation of the SG unit in the case of ten representative days preventing any load curtailment. The overall result indicates a lower cost and less conservative optimal solution with more representative days. On the other hand, both investment and operational costs in the case of SO are shown to increase with an increase in the representative days. Nonetheless, the total costs in the case of DRO are higher than those with SO as the latter provides a more optimistic solution while the former presents a more robust solution.

D. Computational Performance

In Table II, it is indicated that an increase in the budget of uncertainty results in the exponential increment of simulation time of the DRO problem. A similar result is obtained in Table III with more representative days considered in the analysis. Both increments are due to the widening of the uncertainty spectrum captured in the model parameters, i.e., the applied budget of uncertainty, and in the available data, i.e., the representative operation scenarios. However, as compared to the SO model (see Table III), the computational time in the case of the DRO is much greater. A compromise between the data captured and the model parameters must be made to minimise the computational effort.

V. CONCLUSION

In this paper, we have presented a DRO-based MILP planning model for the design of islanded MGs. A moment-based ambiguity is utilised to represent the inherent uncertainty in load and renewable power generation. We propose a three-step approach to reformulate the model into a tractable optimisation problem using LDRs and duality theory. The model is applied to a low-voltage CIGRE network and planning decisions are analysed against the budget of uncertainty, available distributional information modelled by various representative days and additionally compared to the SO model. Future investigations will consider the use of a decomposition solution approach and the variation in type of distributional support provided in the ambiguity set and its impact on the level of conservativeness of the optimal solution.

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